1974

An engineering study of a digital tachometer and its effects on feedback control systems

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AN ENGINEERING STUDY OF A DIGITAL TACHOMETER AND ITS EFFECTS ON FEEDBACK CONTROL SYSTEMS

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THESIS

An Engineering Study of a Digital Tachometer
and its Effects on Feedback Control Systems

by

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) An Engineering Study of a Digital Tachometer and its Effects on Feedback Control Systems		5. TYPE OF REPORT & PERIOD COVERED Engineer's Thesis; September 1974
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Georg W. Goeschel		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, CAlifornia 93940		12. REPORT DATE September 1974
		13. NUMBER OF PAGES 96
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) Naval Postgraduate School Monterey, California 93940		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Digital Tachometer

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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An Engineering Study of a Digital Tachometer and its Effects on Feedback Control Systems

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Submitted in partial fulfillment of the requirements for the degree of

ELECTRICAL ENGINEER

from the
NAVAL POSTGRADUATE SCHOOL
September 1974

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High speed integrated circuitry can be used to realize a tachometer with a high sampling rate at slow speeds. This device would, under certain conditions, be useful in damping transients of fairly fast velocity- and position control systems. After a treatment of the errors which are inherent in this device and other conventional tachometers, a comparative study is made of the effects of digital tachometer on relatively simple feedback control systems. Methods to improve the response are studied.

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LIST OF SYMBOLS

Symbol	Description of symbol
N_R	Number of rotator pulses
n_R	Number of rotator pulses per inch
r_R	Radius of rotator
d_R	Distance between two rotator pulses
f_o	Clock frequency
T	Time between two rotator pulses
n_c	Number of clock pulses, not truncated
n_i	Number of clock pulses, truncated
v	Velocity of circumference, in s
v_m	Velocity measured
v_s	Velocity measured with sampling device
x_T	Distance traveled since last measurement

I. INTRODUCTION

Velocity feedback has been used for many years to compensate control systems. It will make the transient response of second and third order systems more desirable in terms of peak overshoot and damping ratio. The steady state performance for continuously varying inputs, however, will in general be a little worse than without velocity feedback. Fourth order systems will often require additional compensation in the form of filters and will therefore not be treated in this analysis.

Most tachometers used in such systems were either analog measuring devices (Voltage generators) or hybrid digital-analog systems. In the digital-analog tachometer an integration is used, based on counting the number of equally spaced pulses on a transducer in a fixed, predetermined time period, the so called sampling time. The number of counted pulses is then directly proportional to speed. In order to count a sufficient number of pulses, the sampling period has to be rather large (in the order of 10-100 msec), or the number of pulses on the transducer disc has to be very large, imposing difficult manufacturing procedures on the manufacturer. Transducer discs with up to 20000 ppr are available today. The slow sampling rates, however, do not permit the use of these devices for transient analysis or for measuring slow speeds.

Szabados,di Cenzo and Sinha (Ref.1) have introduced a digital device, which is capable of measuring speeds accurately enough from zero to high speeds and can easily be made to measure negative velocities also. With simple modifications it could also be used as an accelerometer.

This new device does not sample over equal time intervals, however, but counts pulses of a fixed frequency during equally spaced angular position intervals. The number of frequency pulses is a direct measure of time. Digital arithmetic devices then divide the known distance by the time measured, and velocity results as output quantity.

This method has values of speed available after each small distance interval, with only a small delay in the arithmetic unit (20 microseconds). Therefore this device is suitable for measurement of transients. To obtain accurate measurements even at slow speeds, a constant rpm "biasing motor" has to be used, which will complicate the physical structure somewhat.

A. OBJECTIVES OF THIS STUDY

The proposed device seems to offer a rather cheap and effective way to measure both transient and slow speeds. It should therefore be possible to use it in control systems as a compensating device, particularly in those applications, where some digital computer is already available to do the counting and the arithmetic.

Since this device samples at different time intervals, depending on velocity, the well known z-transform method of analysis and synthesis cannot be used here, and other methods have to be found.

In order to get more insight into the particular problems associated with the tachometer, an analysis of the errors encountered was done first. These errors are truncation error, due to integer counts, acceleration error, occuring during acceleration between samples, and errors related to the accuracy of the physical structure of the

device itself and the arithmetic unit.

The second objective was to study the effect of the tachometer on feedback systems during dynamic phases. These studies involved second order systems, and parameter changes on the tachometer were used to outline their effects on the system. All models were also studied with analog and sampling feedback, so that excellent comparison was possible. The primary objective was to find a relationship between performance of the system and physical parameters of the tachometer, how bandwidth affects the response and whether the system would eventually become unstable. Later it was attempted to limit or eliminate nonlinear effects by using a first order hold feedback device.

B. METHODS USED IN THIS INVESTIGATION

With the exception of a simple 4-bit arithmetic unit, the total study was done in form of digital simulation on an IBM 360 computer. The simulation of the tachometer proved to be rather difficult, because an integration had to be done in order to obtain the independent variable time, rather than the dependent variable position, which was known in advance in the form of the distance between tachometer pulses. Although good results were obtained, a more accurate and efficient integration would be necessary whenever high speeds are involved.

For the design of appropriate systems, the straightforward Root-locus and Bode plot procedures were used. In order to study stability of a third order system, it was deliberately designed to be unstable without compensation and to have a fairly high natural frequency. The tachometer was then used for compensation.

The following systems were simulated:

First order, Type 0, velocity control
Second order, Type 0, velocity control
Second order, quadratic, velocity control
Second order, Type I, position control
Third order , Type I, velocity control
Third order , Type I, velocity control

The following parameters were changed during simulation:

Number of pulses on tachometer disc (Rotator)
Input amplitude
Poles of transfer function (Bandwidth)
Delay time for sample and hold device
Clock frequency

The results were obtained both on graphs and as printed output and are included, where appropriate, in tables and figures.

II. THE DIGITAL TACHOMETER

A. PHYSICAL PROPERTIES

The tachometer under consideration consists of a low inertia, frictionless disc, called the rotator, which is mounted tightly onto the shaft, the speed of which is to be measured. Equally spaced openings around the circumference of the rotator permit light to travel to a photodiode, which will transmit pulses to a digital unit.

The digital unit consists mainly of a clock (2 MHz used in this study), a counter and an arithmetic unit. The counter is initiated, stopped and reset by the incoming pulses from the tacho disc. The count of clock pulses is used in the arithmetic unit to derive the measured velocity. The count is actually a measure of elapsed time between two tacho pulses

$$n_c = f_o (t_{k+1} - t_k)$$

and velocity is then

$$v_m = \frac{d_R}{(t_{k+1} - t_k)}$$

Since the number of clock pulses depends on velocity, it can be seen that this device will have small errors at slow speed, although the actual resolution capability depends on the number of pulses on the disc.

Most digital tachometers used so far did not use a clock

frequency as a basis to derive time, but counted the tachometer pulses directly over one revolution or over a specific sampling interval. If the sampling interval remains constant, the velocity could be expressed in terms of counted pulses

$$v = \frac{n_c \, 2\pi r_R}{N_R T}$$

and this equation could be used to calculate the number of pulses per revolution required for a given resolution capability at a given speed

$$N_R = \frac{n_c \, 2\pi r_R}{vT}$$

From the equation it can be seen that these devices need many pulses over one revolution for good resolution and small error at slow speed. If the sampling is done at the end of each revolution, the resolution is in the order of one revolution per second, suitable for high speed devices, but not useable as a measuring device for transient analysis.

The activating pulses from the tacho disc have a varying pulsewidth, depending on the actual speed of the device. The counter has therefore to utilize either the rising edge of the pulse or a monostable vibrator must be used to get a pulse with constant width. In the further treatment of the tachometer the varying pulse width is therefore neglected.

The authors of reference 1 and 2 have used an additional bias motor. The photodiodes were mounted on this motor which was rotating with constant speed. In this way a measurement of velocity was possible even at zero speed. With bias speed high enough, negative speeds can also be

measured, however with decreasing accuracy. The velocity equation has to be changed and becomes

$$v_m = \frac{d_R}{(t_{k+1} - t_k)} - v_{bias}$$

Since the counter can only count integers, an error results in deriving the velocity. This truncation error will be treated in more detail in section III. 4 .

In order to use the result of the arithmetic unit, a digital to analog (D/A) converter has to be used, or the result has to be further processed by a digital computer. Errors involving those devices are neglected in this study.

In this study it was always assumed that the linear velocity in inches per second was of interest, rather than the angular velocity in revolutions per second. In most cases the number of pulses per revolution was changed. Since truncation error depends not only on velocity and ppr, but also on the clock frequency, the latter had to be changed occasionally in order to investigate effects of truncation error on a system.

B. TRANSFER FUNCTION

1. Constant Period Sample and Hold Tachometer

In order to point out the differences between the two sampling devices, the derivation of the transfer function for the sample and hold device is done in more detail than actually necessary (Ref.3). For the block diagrams see Fig. 1 and Fig. 2 .

The input to the holding device after sampling is

$$v_s(t) = \sum_{k=0}^{N} v(kT) \quad (t-t_k)$$

The Laplace transform of this is

$$v_s(s) = \sum_{k=0}^{N} v(kT)\, e^{-kTs}$$

After the holding device, the output is a staircase like function and can in time domain be written as

$$v_s(t) = \sum_{k=0}^{N} v(kT) \left[u(t-kT) - u(t-(k+1)T)\right]$$

which laplace transformed becomes

$$v_s(s) = \frac{\sum v(kT)\, e^{-kTs} \quad (1-e^{-Ts})}{s}$$

This can be further reduced, since the sampling time remains constant. The final transfer function is therefore

$$\frac{v_s^*(s)}{v_s(s)} = \frac{\sum_{k=0}^{N} v(kT) \quad e^{-kTs} \dfrac{(1-e^{-Ts})}{s}}{\sum_{k=0}^{N} v(kT) \quad e^{-kTs}}.$$

Or substituting jw for s, the frequency response can be determined

$$G_{Ho}(jw) = \frac{T \sin \frac{wT}{2}}{\frac{wT}{2}} e^{-0.5jwT}$$

This is the well known function for the constant period sampler.

2. Digital Tachometer with Holding Device

If the sampling period is not constant , the last step in deriving the transfer function cannot be performed and the quotient has to be left in summation form.

$$\frac{v_s^*(s)}{v_s(s)} = \frac{\sum_{k=0}^{N} v(t_k)\,\dfrac{e^{-t_k s} - e^{-t_{k+1} s}}{s}}{\sum_{k=0}^{N} v(t_k)\, e^{-t_k s}}$$

For each k, the response of the hold device has to be calculated, making it a very tedious task. Only for constant velocities does this transfer function reduce to the previously derived simple form. (See reference page 346, nonperiodic sampler)

Hufnagel (Ref. 7) has shown in his thesis that for a slowly varying sampling period a special procedure can be utilized to arrive at a reasonable result. This procedure, however, is only valid for slowly varying sampling times. In general systems this restriction cannot be assumed, making the procedure invalid for the purpose of this study. By inspection of the transfer function of the digital tachometer, the frequency response would vary with velocity since the sampling period varies with velocity. The sampling period, however, determines the zero crossings of the frequency response curve.

$$\frac{wT}{2} = n\pi$$

The first zero crossing occurs for n=1

$$w_o = \frac{2\pi}{T}$$

It can be seen that for high velocities the sampling period becomes short, effectively moving the frequency response curve outward. For decreasing velocities the zero crossings move inward, thus increasing or decreasing the bandwidth of the tachometer, depending on speed. Associated with the frequency response is of course the phase curve, which is changing its steepness as the velocity varies. See also Fig. 3 and Fig. 4 .

C. COMPUTER SIMULATION OF TACHOMETER

The computer simulation of the tachometer is based on the only known quantity, the distance between two tachopulses. The program calculates the linear distance traveled by integrating the linear velocity, using discrete steps of time. If the stepsize in time is kept too large, the error in calculating the distance is not acceptable, a large error in clock count would result. This in turn leads to large truncation errors. In order to reduce calculation time, a dynamic integration routine was used, which changes the integration step size, depending on how close the calculated distance traveled approaches the actual distance between pulses. If the integration results in a larger distance, the integration routine restores initial values, reduces the step size and integrates again. This procedure is done until the step size becomes less or equal 10^{-9} seconds. Then the velocity is finally derived and a large step size restored.

In the program, the variable CONCNT is the normalized distance traveled, which truncated becomes actual tachopulse count COUNT. The two variables RCOUNT and LCOUNT provide a means of checking for the first arriving pulse after a start or a reversal of velocity. Only if COUNT1 is equal to two,

does the program calculate the velocity, since the first pulse cannot provide any correct velocity information.

In order to make the tachometer realistic, an initial offset is added in the variable TACSH, taken to be 0.75 in this study. This means, the distance to the first tachopulse after the start of the simulation is 1-TACSH and effectively shifts the velocity measurement in time.

The integration time is the period between two samples. The number of clock pulses counted is then obtained and the measured velocity derived by utilizing the previously defined velocity equation. The program then returns, simulates the system until a new tachopulse arrives and calculates the new measured velocity.

A means of sensing direction is introduced by comparing the last distance with the second last value. The sign of this comparison determines the direction of rotation. In a real tachometer, direction changes could be detected by using a second marker string, displaced in phase with respect to the first string. Depending on which pulse is detected first, a change in direction can be sensed. The program, the flow chart of the complete simulation and a picture of actual versus measured velocity are shown in appendix .

III. ERROR ANALYSIS

A. SAMPLING PERIOD

As previously mentioned, the sampling period for the device under study is a nonlinear function of velocity. In particular, for a constant velocity, the sampling period is

$$t_{k+1} - t_k = \frac{d_R}{v}$$

This nonlinear dependence on velocity is plotted in Fig. 5 , where pulses per revolution (ppr) is a parameter. For low velocities the difference in sampling period for small velocity changes is very pronounced, whereas for high velocities, the sampling period does not change significantly with small velocity changes. However for the same percentage of changes in velocity, the period will vary the same percentage amount, regardless of the initial velocity. This is shown in the example on the next page.

As will be discussed in a later section, the period will also depend on acceleration during sampling intervals. This can be seen in Fig.6 and Fig.7 , where the period is plotted versus velocity, with acceleration as parameter. Fig. 8 shows clearly the drastic variations of sampling period as the velocity changes sinusoidally $v=26+25\sin(wt)$.

Example

Resolution = 333 ppr

Initial velocity = 10 and 50 in/s

Velocity variation= ± 1 in/sec

Velocity (in sec)	Period (sec)	Percent
09	0.0020965	+11.1
10	0.001887	0.0
11	0.007715	- 9.09
49	0.000385	+ 2.04
50	0.000377	0.0
51	0.0003699	- 1.96

Velocity variation= ±10 percent

Velocity (in sec)	Period (sec)	Percent
09	0.0020965	+11.1
10	0.001887	0.0
11	0.007715	- 9.09
45	0.0004193	+11.1
50	0.000377	0.0
55	0.000343	- 9.09

B. ACCELERATION ERROR

In the previous section a constant velocity during a sampling period had been assumed. Since the tachometer was intended to measure transient behavior, this assumption cannot be maintained. Accelerations between sampling periods will occur and should be detected by the device. As in all sampling devices, however, the velocity changes due to accelerations cannot be measured instantaneously. Only at the sampling time can a velocity measurement be made, resulting effectively in a measurement of average velocity. Very fast changes in velocity due to high accelerations might not be detected at all. For the following error analysis a constant acceleration during a sampling period is therefore assumed and seems to be reasonable, at least for high velocities, when sampling periods are short. Since an investigation of truncation error will follow, truncation is not included here for simplicity.

1. Error for constant Period Sampler

A device, which is essentially analog but measures the actual velocity only at specific times, is a sampling device with no error at the time of sampling. The error during sampling intervals depends only on the behaviour of the velocity during these periods (Fig. 9). Any analog to digital converter will have this error, if we neglect quantization error due to digitizing the signal.

2. Error in Sampler counting Pulses

The velocity is now derived from counting marker pulses on the tachometer disc during equal time intervals. The number of pulses counted is a direct measure of distance traveled. This number is dependent on the initial speed at the start of the interval and the acceleration encountered during that interval.

$$x_T = v_o T + \frac{a}{2} T^2$$

The number of markers counted is

$$n_c = x_T N_R$$

The actual velocity is

$$v = v_o + aT$$

whereas the measured velocity is

$$v_m = \frac{x_T}{T} = v + T\frac{a}{2}$$

For this device there is an error during accelerated regions, depending in magnitude on the amount of acceleration and the sampling period.

$$e = v - v_m = \frac{a}{2} T$$

3. Error for Digital Tachometer

This device changes its sampling period with velocity changes and therefore also during accelerations. In principle, the same argument as before holds. See Fig. 11 and Fig. 12 . The number of clock pulses counted depends on the time a certain distance is traveled, in this case the distance between two markers. With initial velocity v_o and a constant acceleration a, the distance traveled is

$$d_r = v_o (t_{k+1} - t_k) + \frac{a}{2}(t_{k+1} - t_k)^2$$

But now the time t is not known and depends furthermore on the initial velocity as well as on acceleration. Since the distance is constant, however, one can solve for the time interval $(t_{k+1} - t_k)$, which will be called Δt for simplicity.

$$\Delta t = -\frac{v}{a} + \frac{v^2}{a} + \frac{2d_R}{a}$$

The dependence on both velocity and acceleration can clearly be seen from this equation. The measured velocity is then calculated to be

$$v_m = \frac{d_R}{\Delta t}$$

Again as in the previous section, the error is

$$e = v - v_m = v_o + a\ t - \frac{d_R}{\Delta t}$$

substituting the equation for Δt, the error can be

expressed in terms of initial velocity at the start of the sampling interval and the acceleration during the interval.

$$e = v_o^2 + 2ad_R - \frac{ad_R}{-v_o + \sqrt{v_o^2 + 2ad_R}}$$

A plot for acceleration error vs velocity with a given acceleration and three different ppr is shown in Fig.10 and Fig.11 . The error reduces significantly for high speeds. Comparing this error with the error derived earlier, it can be seen that the digital device will have less acceleration error.

The acceleration error could also be viewed as a phase shift, particularly in feedback systems. The measurement follows a certain time later than changes may have taken place, thus introducing a phase lag in feedback. This phase lag, while known in a constant period sampler, cannot be predicted for the digital tacho.

4. Truncation Error

In all digital devices, a certain amount of information is always lost due to the quantizing nature of the procedure. In the case of the tachometer, the counter can only count integer numbers. Assuming a square wave clock generator, the counter will be triggered either by the leading or the falling edge. Fractions of cycles cannot be counted. The sign of the truncation error will depend on whether the count starts with the leading edge or the falling edge of the pulse. In one case the count is higher, in the other case lower than the actual fraction would indicate. The error will also depend on whether the clock pulse generator starts with the arrival of the first tacho

pulse, or whether the generator is a continuously running device. In the first case the count would only be truncated at the end of the upcount, resulting in an error of at most one count. In the second case the truncation could occur at the beginning and the end of the upcount, leading to a loss of two counts in the worst case.

The tachometer under study uses very small positional increments for measuring speeds. At high speeds the sampling times are very short. It can therefore be anticipated that a truncation error might become rather pronounced at higher velocities.

<u>example</u>

> For a 2 MHz clock, a 333ppr tacho and a velocity of 50 inch per second, an error of one count (maximum) results in a velocity measurement error of 0.133 % or 0.066313 in/s .

The truncation error is affected by all parameters given in chapter II. It is inversely related to clock frequency and radius of disc, and directly proportional to ppr and velocity.

In the model used, the clock generator was assumed to start with an incoming tachopulse with one count as maximum error. The actual velocity is calculated with distance and total count as

$$v = \frac{d_R f_o}{n_c}$$

The measured velocity is

$$v=\frac{d_R f_o}{n_i}$$

Since the count starts with zero, the measured velocity will therefore be slightly higher, assuming no acceleration during the sampling period. As before, the error is

$$e=v-v_m=d_R f_o \frac{(n_i-n_c)}{n_i n_c}$$

For a count difference of one, a maximum error of

$$e_{max}=d_R f_o \frac{-1}{n_i(n_i+1)}$$

results. The percentage error is

$$e_p=(1-\frac{n_c}{n_i})100\ \%$$

and the maximum percentage error

$$e_{pmax}=-\frac{1}{n_i}100\ \%$$

The last equation indicates clearly the increasing truncation error with increasing velocity (short count).

5. Comparison of Acceleration Error and Truncation Error

These two errors have different signs. While the acceleration error is positive for positive acceleration and negative for negative acceleration, the truncation error is always negative. Moreover, the acceleration error decreases with increasing velocity, whereas the truncation error increases. Thus the total error resulting from these two errors will vary with acceleration and velocity, the truncation error becoming dominant as speeds increase. This can be seen in Fig. 15 , where the error of slowly increasing velocity ramp (acceleration=320 in/s) is shown.

A calculation of velocities in the vicinity of negative errcrs clearly shows the effect of truncation.

Example

Velocity in in/s		
actual	: 60.424058	60.623579
measured without truncation	: 60.374070	60.573776
measured with truncation	: 60.378957	60.673374
Error=$v-v_m$ in in/s		
without truncation	: 0.049988	0.049816
with truncation	: 0.045100	-0.049794
Number of cycles counted	:625.05	622.988

In Fig. 16 , the same velocity was measured without truncation, resulting in an even distribution of the error. The decreasing width of the error curve indicates a decrease in acceleration error since the sampling periods become shorter. In preparing the last two curves, a bias motor with 50 in s was used.

IV. USE OF THE TACHOMETER IN FEEDBACK SYSTEMS

A. GENERAL CONSIDERATIONS

The most common form of analysis and synthesis in linear systems involves the use of a transfer function, which describes the behavior of the system for a unit impulse function. It has been shown that a feedback-sampled system cannot be reduced to a simple output vs input transfer function, not even with with z-transform methods (Ref. 3 page 222). The output function for a feedback-sampled system would in general form be

$$C(z) = \frac{GR(z)}{1+GH(z)}$$

where $GR(z)$ is a product and cannot be factored. It is because of this that feedback-sampled systems received little attention so far. A general procedure is therefore not obtainable and computer simulation for a particular system becomes very important.

The original intention of this study was to investigate the influence and effects of a digital tachometer on a rather complex, reel to reel tape drive system, which uses this device as velocity and position control for tape motion. Because of improper modeling of tape tension and tachometer wheel, the tachometer and its effects could not be studied, high frequency oscillations obscured the effects of the tacho. Due to time limitation , the attempt had to be abandoned and a simpler system sought. In pursuing a more general type of tachometer analysis, it became more important to investigate the critical region of the tachometer, i.e. the effect of reducing the ppr more and

more until the device would become useless. In the limiting case, when only one pulse per revolution is used, the device is the same as the one that measures the time of one revolution in order to arrive at a speed measurement. Increasing the number of pulses on the rotator makes the device more and more compatible with a normal analog feedback device and the usual methods of analysis could be used. To get meaningful results for the simulation of transients, the systems which were to be simulated had to have reasonably high natural frequency and large bandwidth. A frequency in the order of 50 Hz was thought to be typical in electrical motors and servos. This assumption met with the values which were used in the original paper from the authors previously mentioned.

B. VELOCITY CONTROL

1. First order System

From the block diagram of the originally investigated tape drive, a linear version of one of the drive motors provided a simple transfer function

$$G(s) = \frac{23}{s(s+0.25)}$$

which was used in a first order velocity control system (Fig. 17), calling for a 25 in/s nominal speed, or equivalent of 1.2 to 1.6 rev/s for a tape reel of 6 in average diameter. This system is inherently stable without overshoot. Modifying the block diagram as in Fig. 18 and looking at the error

$$e = R - C_f$$

one can use the type zero error coefficients tc determine

the steady state error for an analog system and compare this with results obtained from runs with a digital tacho.

Comparing the transient region first, however (Fig. 19), it can be seen that during the starting phase the velocities in the system with digital feedback are slightly higher, since feedback lags behind actual velocity. The start is even faster than in a system employing constant period sampling with T=0.01 seconds. But since the digital tacho decreases sampling time as velocity increases to 25 in/s, the response of that system resembles more closely that of the analog system as it approaches higher velocities. For a ppr=167 and velocity=25 in/s the sampling period is 0.0015 against 0.01 for the sampler. A digital tacho with the same sampling period at 25 in/s would only need 25 ppr. Such a system would have a steep transient curve however. To obtain fairly good resolution in a constant period sampler, a ppr of about 2000 would be necessary, which would result in a measurement of about 80 pulses per sample at the nominal speed of 25 in/s. The truncation error is then still larger than 1 %, and would further increase for slower velocities.

<u>Some comparative numbers</u>

Time	analog	digital	sampled
0.01	5.153	5.748	5.747
0.02	9.200	10.420	10.156
0.03	12.422	13.636	13.540
0.04	14.976	16.066	16.140
0.06	18.603	19.420	19.666
0.07	19.875	20.650	20.842
0.10	22.313	22.717	22.970
0.20	24.495	24.547	24.606
2.00	24.731	24.700	24.731

The shape of the transient depends also on the initial offset of the tacho. If the first arriving pulse could already be used for velocity measurement, then the response would not be as steep as it is with a longer delay time at the start. Until the first measurement, the system acts as if there was no velocity feedback at all. Then the velocity feedback sets in, however with smaller measured velocity than the actual velocity. The difference between the measured and true velocity is decreasing as the system approaches nominal speed. This means the system acts as if the feedback possesses a variable gain factor, starting with zero and approaching one as velocity approaches nominal velocity and accelerations go to zero.

The analog steady state error is calculated to be 0.2688 in/s for a step input of amplitude 25 and infinite for a ramp input.

If a digital tachometer with the same gain constant would be used, then the previously defined errors have to be added. For a step input, only the truncation error adds to the steady state error. For high velocities the truncation error results in a velocity measurement higher than true velocity and the error becomes even negative. This results in an even larger steady state error than for an analog system. To illustrate this effect, a large truncation error was deliberately created by using a large number of pulses (666) and a low clock frequency (0.2 MHz), the justification for these numbers being shown below in the derivation.

The maximum truncation error was previously found to be

$$e_{max} = \frac{2\pi r_R f_o (-1)}{N_R n_i (n_i + 1)}$$

but we also know

$$\frac{2\pi r_R f_o}{N_R (n_i+1)} = 25$$

and the error can be expressed as

$$e_{max} = \frac{-25}{n_i}$$

or with a given error

$$n_i = \frac{-25}{e_{max}} = 93$$

and then with f=0.2 MHz

$$N_R = \frac{2\pi r_R f_o}{e_{max} n_i (n_i+1)} = 534.7$$

The steady state error which now resulted was 0.494 in/s.

Count variations due to truncation might even result in a limit cycle at the output. Final velocity as well as the occurance of a limit cycle depends on the dynamics of the system. In some of the velocity control systems and in all position control systems a limit cycle was actually observed and will be pointed out in the discussion of those systems.

The added error from the digital tachometer will become more and more dominant for a ramp input, since in this case the velocity increases and the truncation error will become rather large. This can be shown easily using time domain analysis.

For a ramp input of 1000*t, the function for velocity is

$$C_f(t)=42.548(e^{-23.25t}+23925t-1) \quad in/s$$

the error for an analog system after one second is

$$e=1000-946.699=53.3 \ in/s$$

For a digital tachometer with 666 ppr and 2 MHz clock frequency, a maximum error of

$$e_{max}=1000-v_m=3.25 \ in/s$$

would result. It is assumed that acceleration error becomes negligible at these speeds. After two seconds the maximum error due to truncation would even be -129.6 in/s instead of +106.6 in/s due to steady state error. This would result in a lower velocity at the output and the error would increase faster than in an analog system.

The last calculations and results show clearly three facts. First, the feedback gain is variable during transients, due to acceleration error, which depends on the input amplitude. Second, truncation error may become rather large and dominant, and third, it is better to use a small number of tachopulses for high speeds, because this reduces the truncation error. A steady state error analysis in the conventional form as used in analog , unity feedback systems cannot be used whenever high velocities are involved.

2. <u>Second order System</u>

A first order system did not present any difficulties, measured as change of settling time, overshoot and error resulting from the utilization of the digital tachometer. As a slightly more complicated system, a second order, well damped unity feedback system was chosen. Effects of sampling were expected to be more noticeable in such a system. The open loop transfer function was selected to be

$$G(s) = \frac{100000}{(s+100)(s+1000)}$$

For a step input, the system exhibits a fast rise time, no overshoot with analog feedback and a steady state error of 0.5. As before, the speeds were limited to rather small values, 50 in/s and 25 in/s respectively as nominal speed for comparative runs.

The response to a constant period sampler became underdamped for sampling times larger than 0.005 seconds, and an overshoot of 3.8 % occurred for a sampling period of 0.006 s, regardless of the amplitude of the input signal. The slightly larger velocities at the start resulted in a larger distance traveled, which would be important in position control devices.

With the digital tachometer feedback, the following observations could be made:

1. The steepness of the startup was increased and was also dependent on the final velocity.
2. An overshoot occurred for small ppr and large sampling intervals.
3. The overshoot vanished for high inputs and a ppr

larger than 167.

4. A small limit cycle was observed and the final velocity was slightly smaller, indicating a larger steady state error.

Since the rise time of the system was rather fast, the amount of overshoot will be affected by the amount of initial offset of the tachometer. When a ppr of 42 was used, only two measurements can be taken until time of peak overshoot, after that the system is brought to nominal speed and slow velocity changes can be detected quite easily.

A comparison of the responses to an analog feedback, a sample and hold feedback and a digital tachometer feedback can be seen in Fig. 20 . Numerical results are listed in table I, where the settling time is taken to be the time until the response is within 2 % of the nominal velocity.

In order to study effects of the digital tacho in systems with rapid velocity changes, the well damped second order system was changed into an underdamped system. The open loop transfer function became now

$$G(s) = \frac{100000}{(s^2 + 189.7s + 100000)}$$

which has a damping coefficient of 0.3, a natural frequency of 50.3 Hz and a corresponding settling time of $4\tau = 0.042$ seconds. With the loop closed, rho becomes 0.212 and w=447.2 . As before, similar observations as in the well damped system were made. Unstable conditions resulted for sampling periods of 0.005 seconds or larger in the case of a constant period sampler and for ppr of 83 and less with the digital tacho. A ppr of 83 results in a sampling period of 0.0015 seconds at a velocity of 50 in/s . For the digital tacho, the response to a step input will become better and

better as the pulses on the disc increase, the specific response depends on the amplitude of the input however. A small limit cycle was observed, which was to be expected, since the system is underdamped and truncation error does not permit the system to stabilize itself. Initial offset at the start does not affect the response in any noticeable way.

For a ramp input, the response of the digital tacho feedback system is not nearly as good as for the analog system or even as for the constant period sampler. At the start, the velocity is slow and only very few measurements will be taken. This resulted in a rather oscillatory start until at higher speed the sampling rate increases to acceptable values and the system behaved like the other two systems. A plot of the response to a ramp input is provided in Fig. 23 , where this effect can clearly be seen. In Table II, the results of a second order quadratic system are listed for step inputs.

3. Third order system

Adding a pole at zero resulted in a third order system and the gain was first chosen to keep the system slightly underdamped.

$$G(s) = \frac{10000000}{s(s+100)(s+1000)}$$

The nominal speed was 100 in/s and 50 in/s , which permits rather high sampling rates at full speed and a reasonably good result could be expected. From Fig. 24 it can be seen that for a ppr of 167 and larger, the response is close to the analog response. Because the system was stable under closed loop conditions, it was hard to reach large overshoots, even with ppr as low as 42. For a constant period sampling device, a sampling rate of 1000 samples per

second would be necessary to achieve the same response, and for good accuracy a ppr of about 6000 would be needed.

With the gain increased to $5*10^7$, a very poorly damped system resulted, requiring a faster sampling rate for good response. With 42 or 83 ppr on the tacho disc, the response was virtually unstable, although the oscillations did not increase amplitude after they reached a final value. As can be deduced from Fig. 25 , a reasonable response is only achievable with a large number of pulses. A limit cycle occurs and is larger than in other cases because the system dynamics are much more unstable.

In order to reduce truncation error, a change in clock frequency was also investigated. A frequency of 20 MHz did affect the response only minimally. The peak overshoot which resulted was 72.8 % compared with 72.5 % with larger truncation error. But since the error is negative it reduces the overshoot. A further reduction in clock frequency to 0.2 MHz produces even larger truncation error. With a ppr of 666 , the resulting count at full speed is only about 20,which means a 5 % error when one count is lost. In the system, this large error caused only 69.15 % overshoot, a considerable reduction compared to the previous runs. Again, a listing is provided in Table IV .

As pointed out in an earlier chapter, a possible way to improve the digital tachometer is to replace the holding device by a first order hold, which would integrate between samples according to the rate of change of measured quantity between the two latest samples. The equation of measured velocity is than

$$v_m = v_k + \frac{(v_{k-1} - v_{k-2})}{(t_{k-1} - t_{k-2})} * (t - t_{k-1})$$

Table IV includes the results of a run with a first order hold in the feedback path, and an improvement can be observed when comparing the Fig. 26 with Fig. 25 . The improvement did not change the settling time, however. This can be attributed to the occurrence of the limit cycle. The response to a ramp is very poor at the beginning as in the second order system.

The amount of overshoot depends not only on the characteristics of the plant, but also on the parameters of the feedback device. A plot of peak amplitude vs ppr for two particular systems is shown in Fig. 27 and Fig. 28 . A definite relationship to the input amplitude can clearly be seen. Although for each input a new curve would have to be drawn, intermediate values are easy to interpolate from the curves available. This was actually done and the result was as predicted. Finally it can be said that all velocity control systems had a slightly faster start when using the digital tacho, and the peak time was slightly less than with either analog or constant period sampling feedback.

C. POSITION CONTROL

1. Second order System

In a position control system, the tachometer is used for compensation in case the response would be too underdamped or even unstable with unity feedback only. The third order system chosen later was therefore designed to be unstable without the tachometer compensation.

The same arguments concerning rise and settling time as with velocity control systems apply and the first system was a second order, type I, unity feedback system with transfer function

$$G(s) = \frac{111100}{s(s+100)}$$

which then used tachometer feedback with a gain of 0.0009. The total transfer function became then

$$G(s) = \frac{111100}{s^2+200s+111100}$$

with a natural frequency of 53 Hz and a rho of 0.3, which results in a peak overshoot of 37.2 % in an analog system. In Fig. 29 , the response of various types of velocity feedback to a step input is shown. The velocity will eventually go to zero, and a plot of velocity vs time for this system is included in Fig. 30 , where the smooth curve is the actual velocity and the staircase curve the measured velocity. The following observations are possible on this simple system:

1. The peak amplitude with digital feedback is slightly smaller than in the analog system for all values of ppr chosen, while for the constant period sampler the amplitude increases.
2. The bandwidth increases as can be seen by the increased natural frequency. This effect was predicted in chapter II. The frequency changed to 59.5 Hz with ppr of 83 and to 53.7 with a ppr of 666, approaching the response of the purely analog system without tachometer feedback as the position reaches the final value.
3. The response exhibits long ringing (limit cycle), since at steady state no velocity feedback exists, as can be seen in Fig. 30 .

In analyzing the phenomena observed when digital feedback is used, the tacho gain factor has to be considered variable. It will vary from zero at the start to the chosen

value for analog velocity feedback and even to a value larger than the chosen one when the acceleration is negative, because of the phase lag in measurement. The larger gain factor could account for the reduced peak amplitude. The peak occurred when the velocity changes direction, i. e. after the system went through a period of negative acceleration. Since the acceleration error depends on initial velocity at the instant of sampling and on the ppr, its effect should be reduced whenever the systems dynamics involve high velocities. In a positioning system, velocities are a function of input amplitude. Therefore the percentage overshoot should also be a function of input amplitude for a digital feedback system. Only with high velocities will the truncation error affect the response.

2. <u>Third order Position Control System</u>

In order to study the observed effects more thoroughly, an originally unstable third order system was compensated with a tachometer which used a gain of 0.00215. The transfer function became then

$$G(s) = \frac{1.5\ 10^{8}}{s^{3} + 1100s^{2} + 422500s + 1.5\ 10^{8}}$$

with open loop poles at zero, 100 and 1000, and a natural frequency of 58 Hz. The previously mentioned dependency on input amplitude was now confirmed by using several input sizes. Table V contains a listing of peak amplitude and limit cycle amplitude for various step inputs. It can be seen that the response depends on input amplitude and, as was the case in the second order system, the peak became actually smaller than the analog systems overshoot for inputs larger than one and virtually all ppr values selected. For small inputs, the response must be considered unstable, since violent oscillations occur with amplitudes

comparable to the input step size. It was interesting to note almost the same amount of overshoot at 4 times the input amplitude with 1/4 th of the ppr, as seen with ppr of 83 and input 2.0 vs a ppr of 333 and input 0.5 inches. Since the system was unstable without tachometer, a limit cycle resulted, because at steady state the feedback gain reduces to zero for a step input. This limit cycle is independent of input amplitude and depends on ppr and the systems dynamics. Its amplitude was about 0.04 in with a ppr of 333 and 0.15 in with a ppr of 83, not exactly a 4:1 relationship. The frequency of the limit cycle is again slightly higher than the natural frequency. Fig. 31 and Fig 32 show the position and velocity response to an input of 1.0 inch.

In order to find the frequency response curve, a series of runs was made with several frequencies and different tachometer parameters. Numbers are presented in Table VI, while a plot of two frequency curves is shown in Fig. 33 . It can be seen that the frequency response curve is not only shifted to the right (increased bandwidth), but also changes in gain. While the gain is lower for frequencies less than the natural frequency, it increases for higher frequencies until it falls off. It has to be indicated again that during a particular transition the frequency response is dynamically changing, depending on the form of the input, i. e. differently for a step input, a ramp or other input functions.

For a ramp input, this system behaves like a stable velocity control system with a step input, and a treatment of ramp input response is therefore omitted. An attempt to improve the response by using a first order hold feedback was also made. But while the transient resembles now more closely the response of an analog system, the steady state response is worse in that the the limit cycle becomes larger in an irregular way, as can be seen in Fig. 34 and Fig. 35 .

V. CONCLUSION

The proposed tachometer has characteristics which are superior to all velocity sampling devices previously used. If carefully designed, it will be helpful in compensating velocity and position systems. It is important, however, to choose the parameters of the tacho according to the nominal operating region of the system, since the output of the tacho depends on input amplitude. As the authors of Ref. 1 point out, a bias motor will greatly improve the response because it allows measurement even at zero velocity. Since the sampling rate is very high, an analog system analysis should in most cases be justified.

Under certain conditions and with careful design, it may be possible to use the truncation error as an aid to damping. This had been demonstrated in chapter IV. C 2 .

It is furthermore easy to use a second counter to count the number of tachopulses since the start, which then provides position information as well.

Whenever high velocities have to be measured, a very high clock frequency would have to be used in order to reduce the truncation error. If, however, a reasonable number of tachopulses could be utilized, then one can divide the full operating region into two portions, using the proposed tachometer at startup and slow velocities, while switching to a regular sample and hold tachometer at high speed. The sample and hold device would then still get enough pulses to provide good resolution.

Another possible application of the tachometer uses previous measurements to obtain an estimate of acceleration.

Since the digital tacho provides data at a fairly high rate, the acceleration measurement should be rather accurate and the obtainable resolution superior to the sampling and counting routines used so far.

From the previous conclusions it can be seen that the use of the digital tachometer leaves many areas to be studied. The author believes that further study of the mathematical analysis is necessary. Another suggested field of investigation is the actual design of the arithmetic unit with the newest integrated circuits available. An interesting aspect of this would be the possible use of a microprocessor as a control unit and, may be, even as the arithmetic unit itself. It is felt that with the reduction of prices for single units and the incorporation of memory into the processor, a fairly cheap but accurate digital tachometer could be built.

TABLE I

DAMPED SECOND ORDER SYSTEM

	time of peak	% overshoot	settling time
analog			0.0175
sampled			
delt=0.001			0.0175
delt=0.003			0.013
delt=0.005	0.0131	3.96	0.015
digital			
	input=25 in/s		
ppr= 42	0.0141	25.84	0.0036
ppr= 83	0.011	17.6	0.0195
ppr=167	0.014	0.452	0.0098
	input=50 in/s		
ppr= 42	0.011	10.65	0.0191
ppr= 83	0.012	1.8	0.0085
ppr=167			0.015

TABLE II

QUADRATIC SECOND ORDER SYSTEM

	time of peak	% overshoot	settling time
analog	0.00718	54.2	0.042
sampled			
delt=0.005	0.0071	101.2	long
delt=0.003	0.0071	83.0	0.152
delt=0.001	0.0071	62.0	0.058
digital			
	input=25 in/s		
ppr=167	0.00708	91.7	0.166
ppr=333	0.00704	73.9	0.072
ppr=666	0.00705	63.2	0.052
	input=50 in/s		
ppr=167	0.00701	73.8	0.060
ppr=333	0.00709	63.0	0.051
ppr=666	0.00707	57.0	0.044

TABLE III

DAMPED THIRD ORDER SYSTEM

	time of peak	% overshoot	settling time
analog	0.0361	20.38	0.085
sampled			
delt=0.006	0.0351	28.80	0.110
delt=0.003	0.0351	38.35	0.143
digital			
	input=100 in/s		
ppr= 42	0.0330	36.03	0.177
ppr= 83	0.0340	28.9	0.086
ppr=167	0.0351	24.4	0.086
ppr=333	0.0355	22.4	0.085
ppr=666	0.0360	21.36	0.085

TABLE IV

UNDERDAMPED THIRD ORDER SYSTEM

	time of peak	% overshoot	settling time
analog	0.0151	69.11	0.162
sampled			
delt=0.001	0.0151	75.20	0.2+
delt=0.0005	0.0151	81.54	long
digital			
	input=50 in/s		
ppr= 83			
ppr=167	0.01505	99.40	long
ppr=333	0.01503	85.10	0.2+
ppr=666	0.01507	77.00	0.19
	input=100 in/s		
ppr= 83	0.01504	102.60	long
ppr=167	0.01504	85.17	long
ppr=333	0.01506	77.06	0.2+
ppr=666	0.01504	72.50	0.177

with first order hold, input=100 in/s

ppr= 83	0.01501	95.40	0.2+
ppr=167	0.01505	81.04	0.2+
ppr=333	0.01502	71.47	0.17

with clockfrequency 0.2 MHz, input=100 in/s

ppr=167	0.01502	8411	long
ppr=333	0.01509	75.11	long
ppr=666	0.01502	69.15	long

TABLE V

THIRD ORDER POSITION CONTROL SYSTEM

input	% overshoot		% limit cycle	
analog	27.34			
digital	ppr=333	ppr=83	ppr=333	ppr=83
0.25	28.98	46.34	13.77	65.956
0.50	27.76	34.10	6.40	29.800
1.0	27.39	29.51	3.84	15.57
1.5	27.16	28.39	2.8	9.325
2.0	26.85	27.90	1.8	8.18

TABLE VI

FREQUENCY RESPONSE OF THIRD ORDER SYSTEM

frequency	amplitude in dB			
	analog	ppr=333	ppr= 83	ppr= 42
20	0.445	0.366	0.121	-0.087
30	0.867	0.765	0.473	0.374
40	1.756	1.613	1.356	0.668
50	2.652	2.542	2.280	2.007
60	3.020	3.033	3.107	3.106
70	2.130	2.245	2.866	3.522
80	-0.028		1.214	2.212
90	-2.722	-2.270	-1.7170	-0.354
100	-5.449	-5.036	-3.835	-2.854
110	-7.851	-7.535	-6.410	-5.680

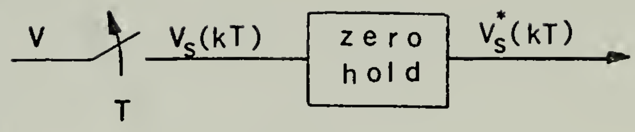


Figure 1.
Constant period sample and hold

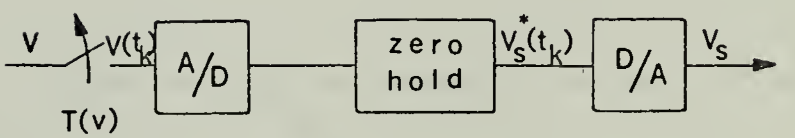


Figure 2.
Digital tachometer sample and hold

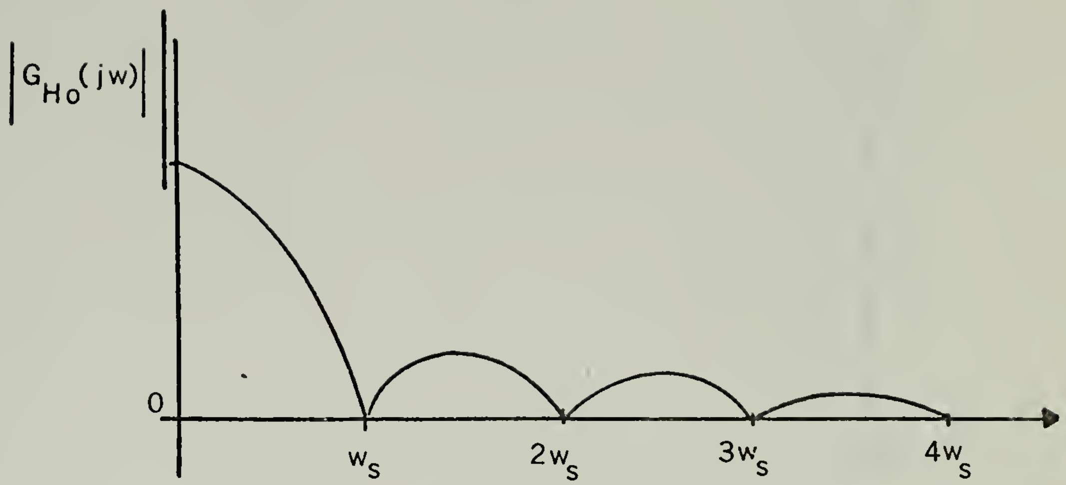


Figure 3.
Frequency response curve for sample and hold

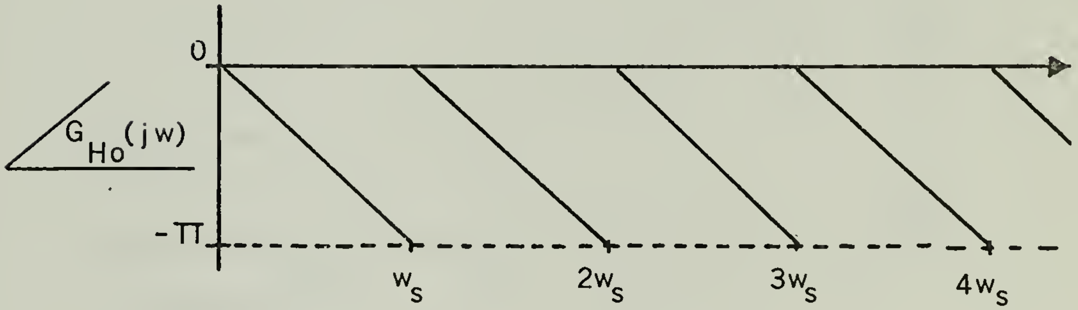


Figure 4.
Phase response curve for sample and hold

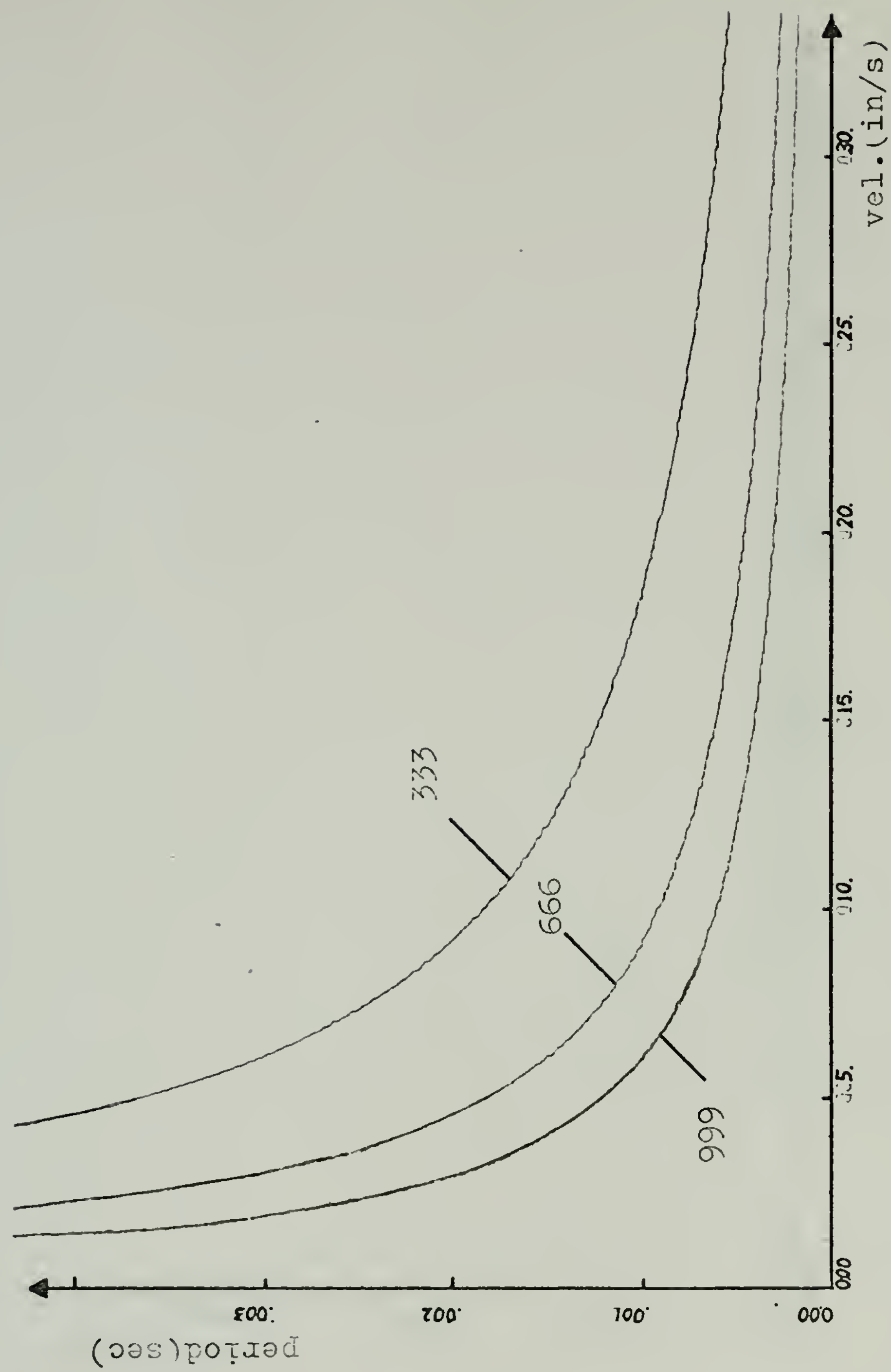
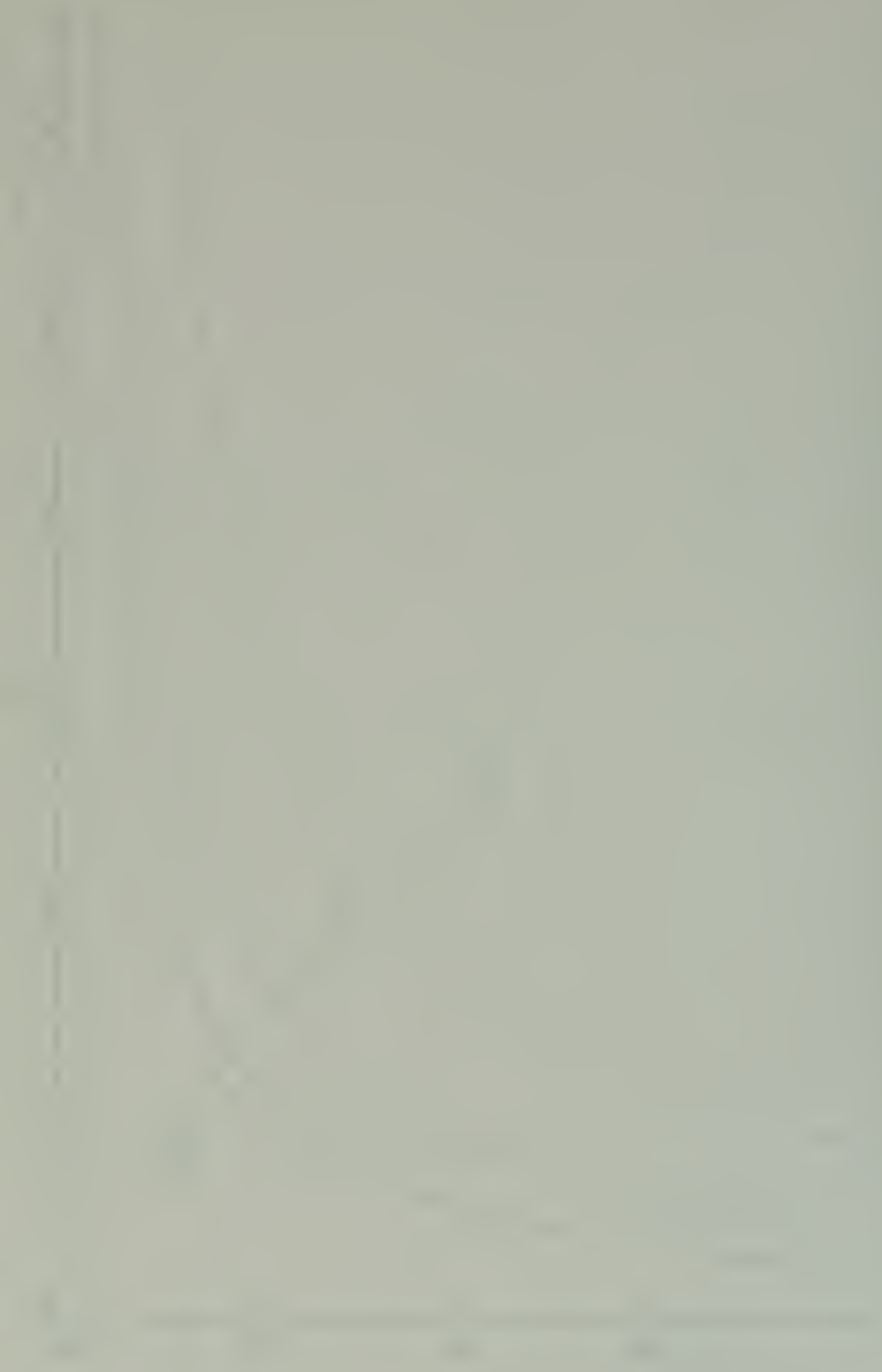


Figure 5.
Sampling period vs velocity
ppr=333,666,999 no acceleration



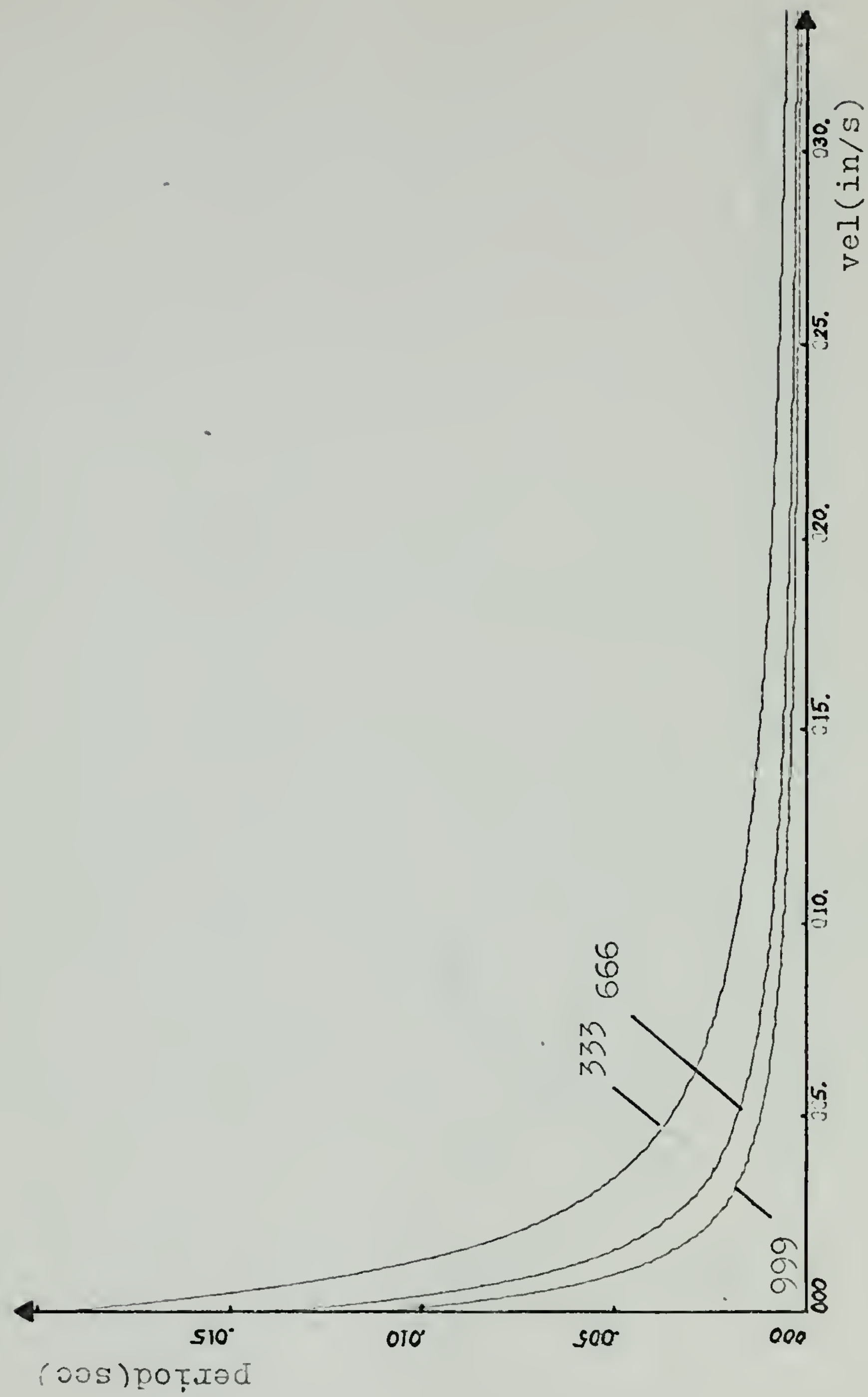


Figure 6.
Sampling period vs velocity
ppr=333,666,999 acceleration=100 in/s

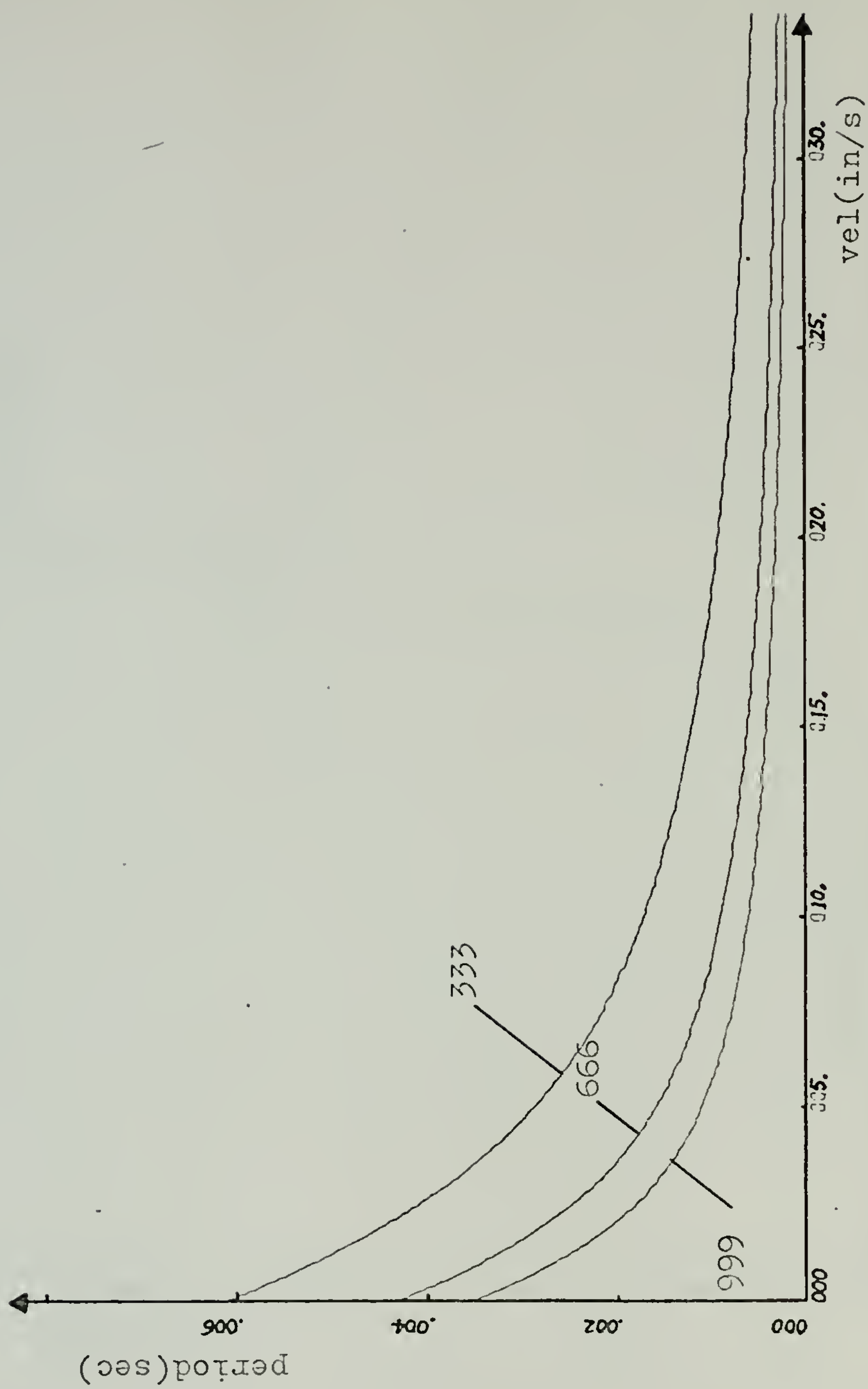


Figure 7.
Sampling period vs velocity
ppr=333,666,999 acceleration=1000 in/s

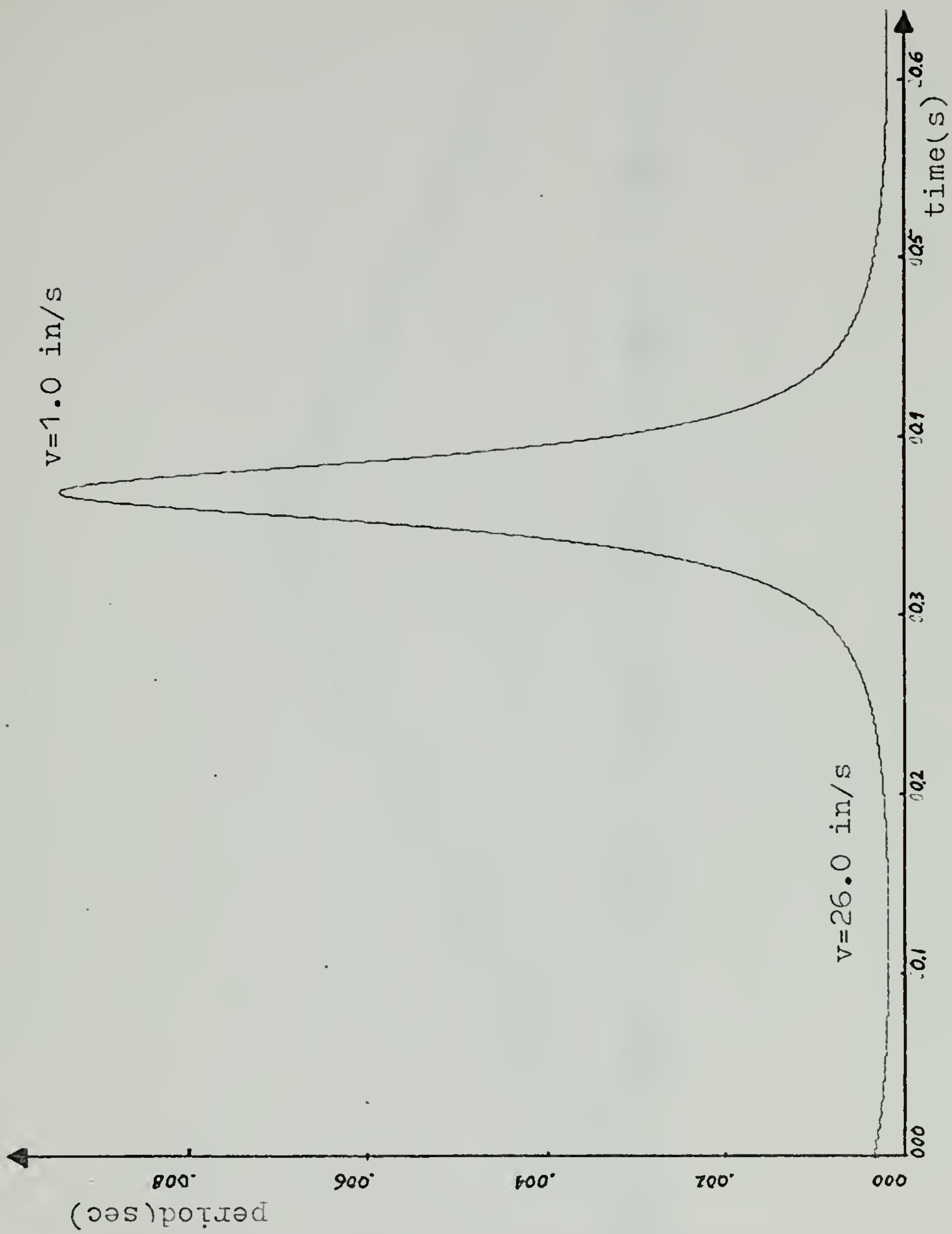


Figure 8.
Sampling period vs time
velocity=26+25sin(w*t)

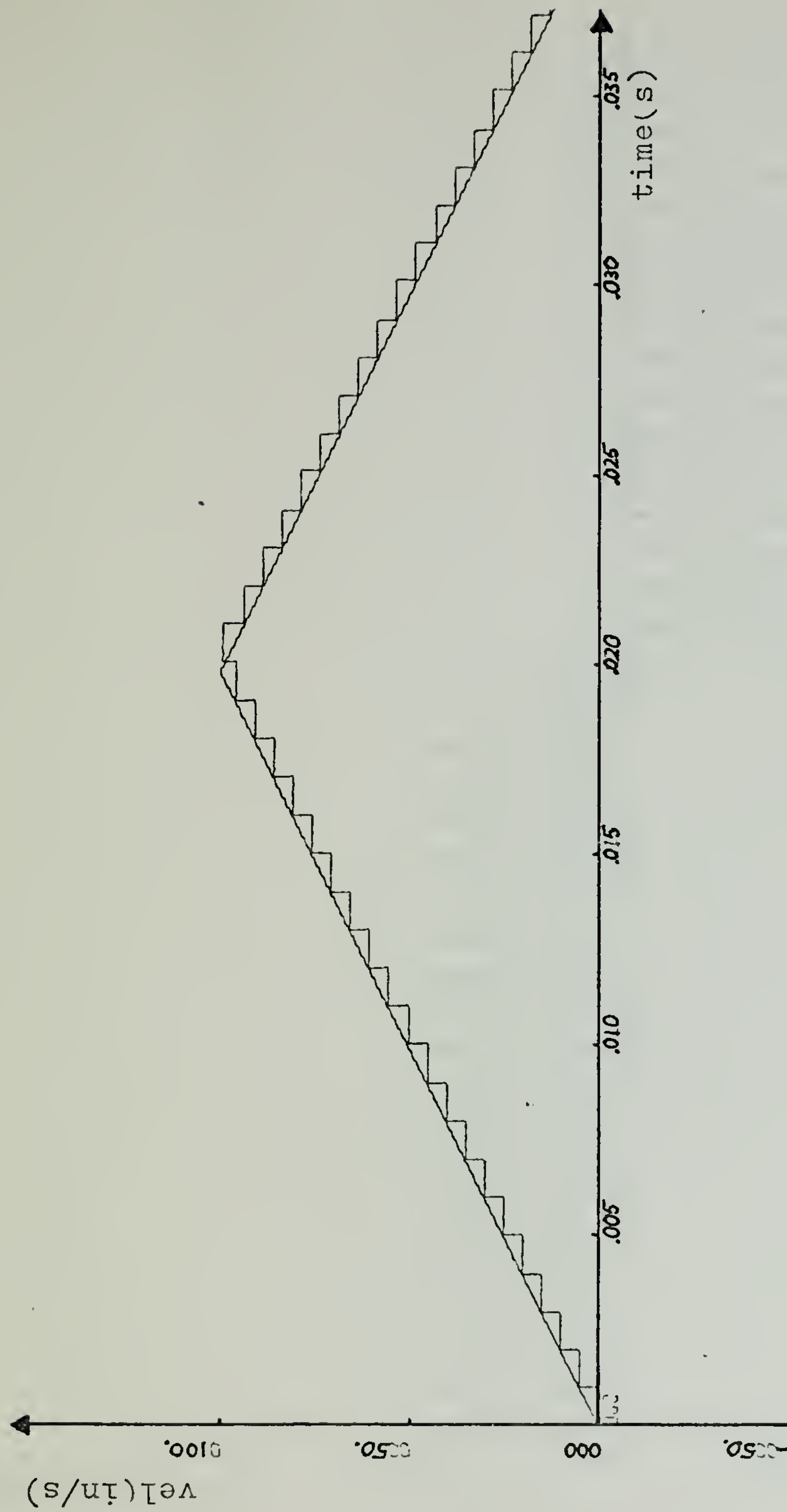


Figure 9.
Constant period sampler,delt=0.001
actual and measured velocity vs time

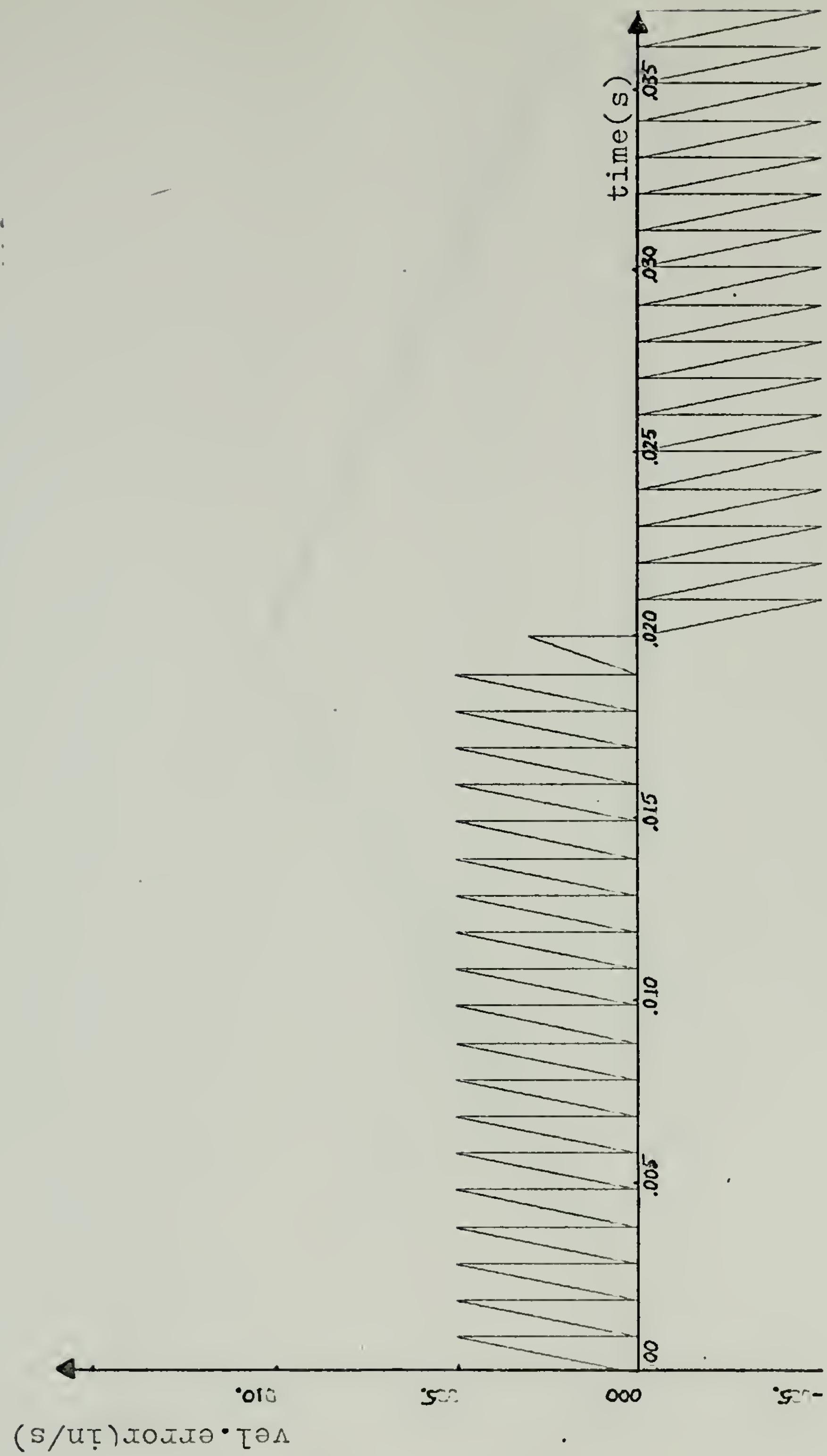


Figure 10.
Constant period sampler,delt=0.001
error vs time

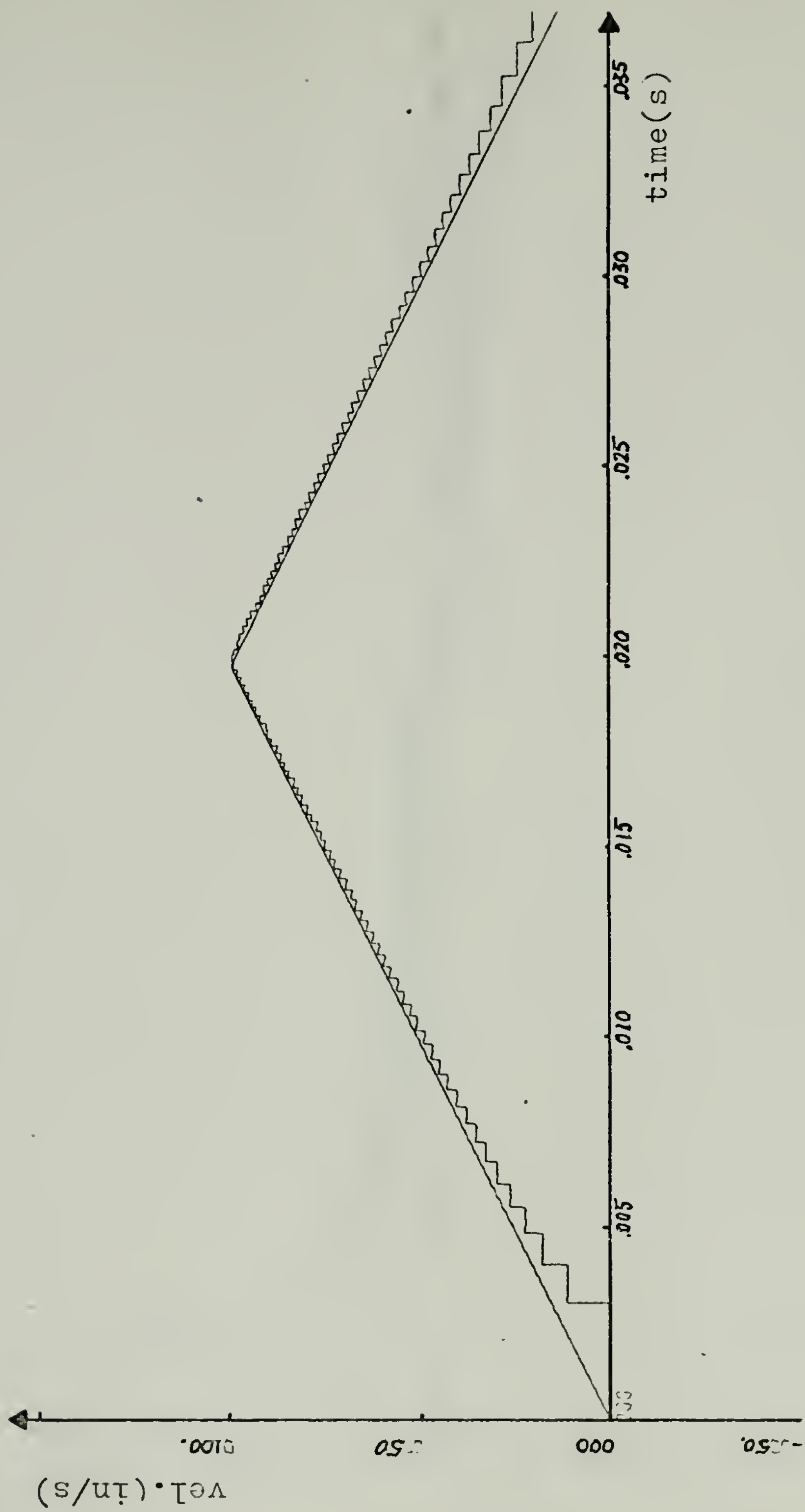


Figure 11.
Digital tachometer, ppr=333
actual and measured velocity vs time

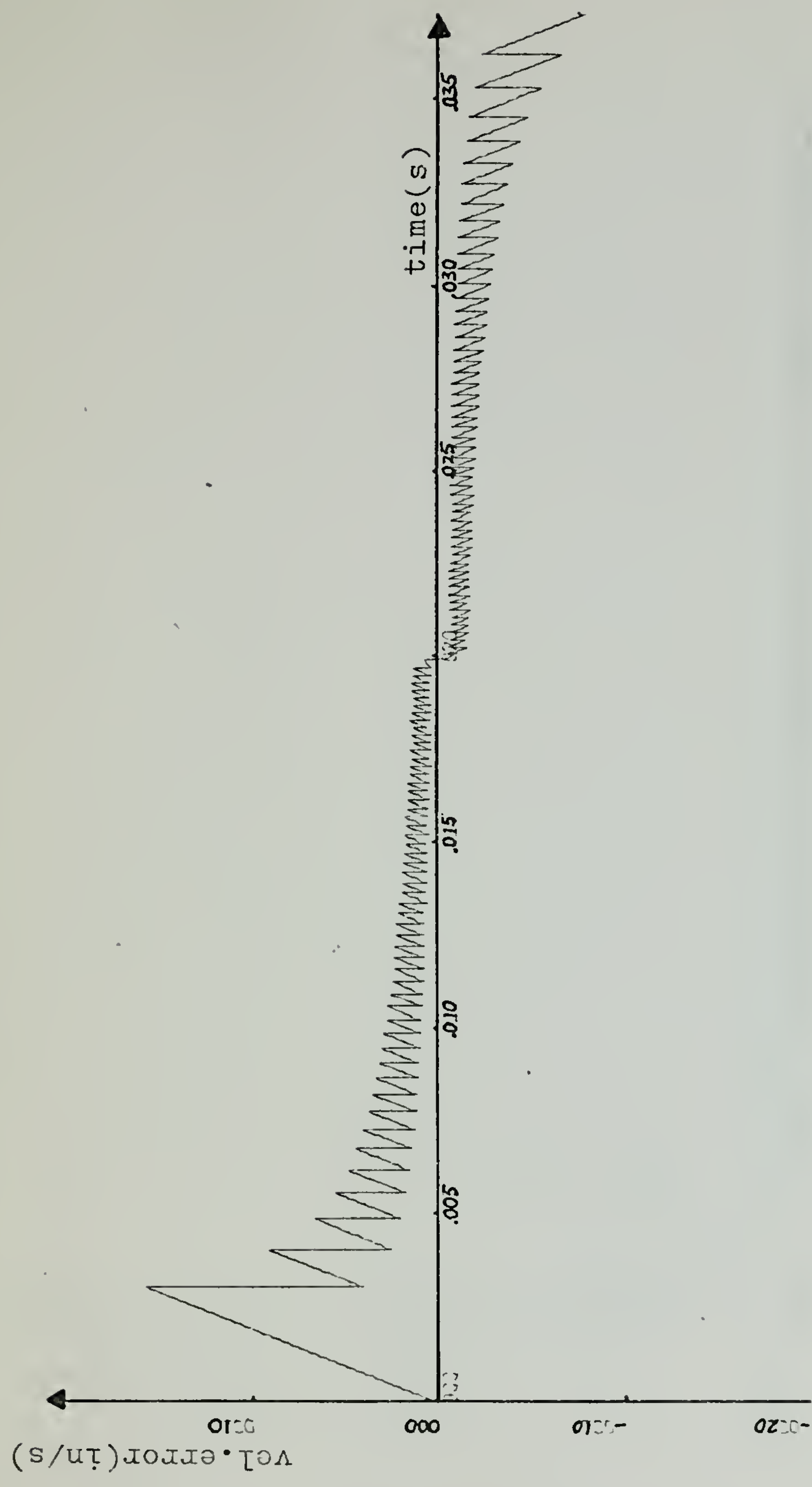


Figure 12.
Digital tachometer, ppr=333
error vs time

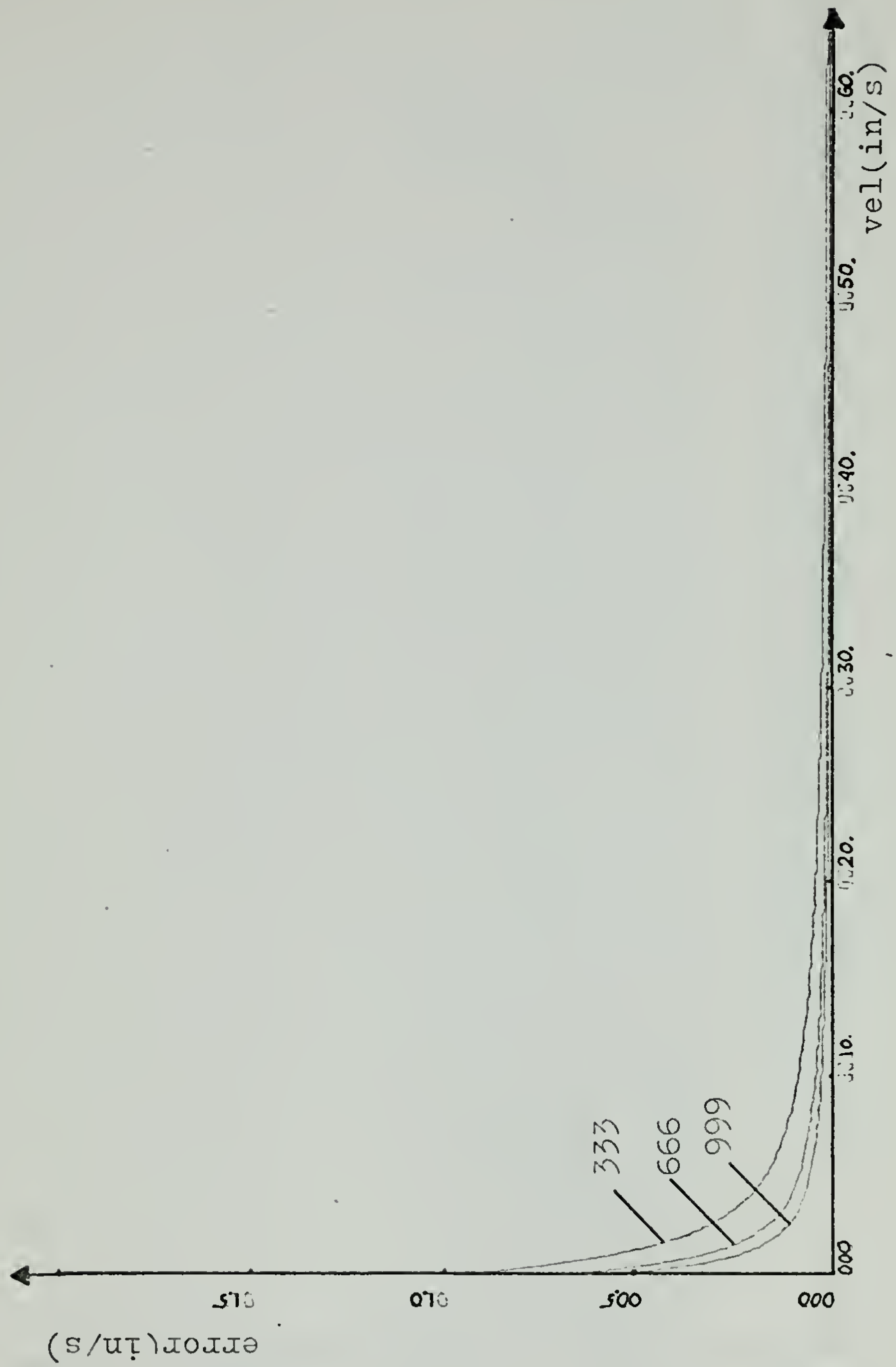


Figure 13.
Acceleration error vs velocity
ppr=333,666,999 acceleration=100 in/s

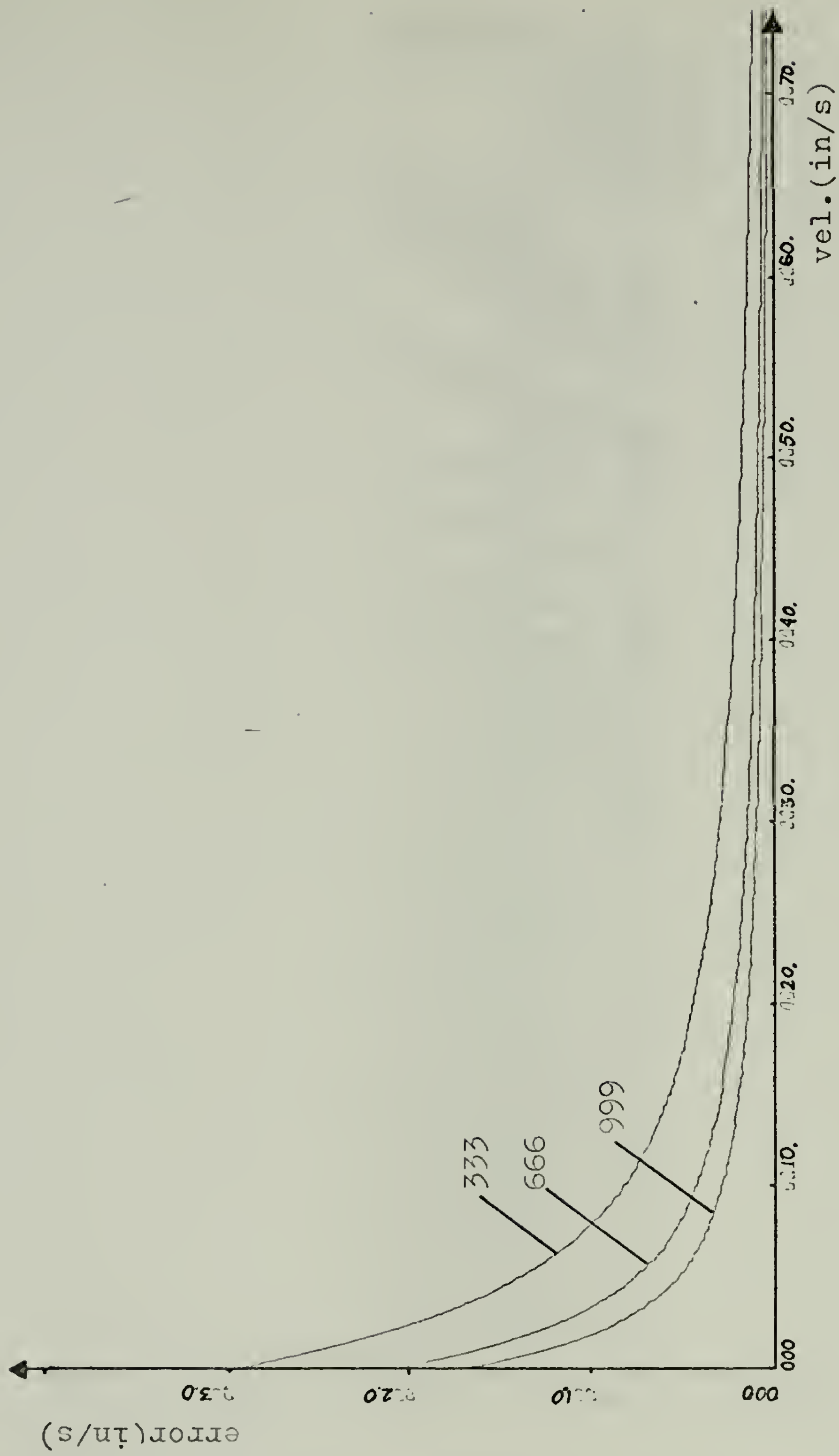


Figure 14.
Acceleration error vs velocity
ppr=333,666,999 acceleration=1000 in/s

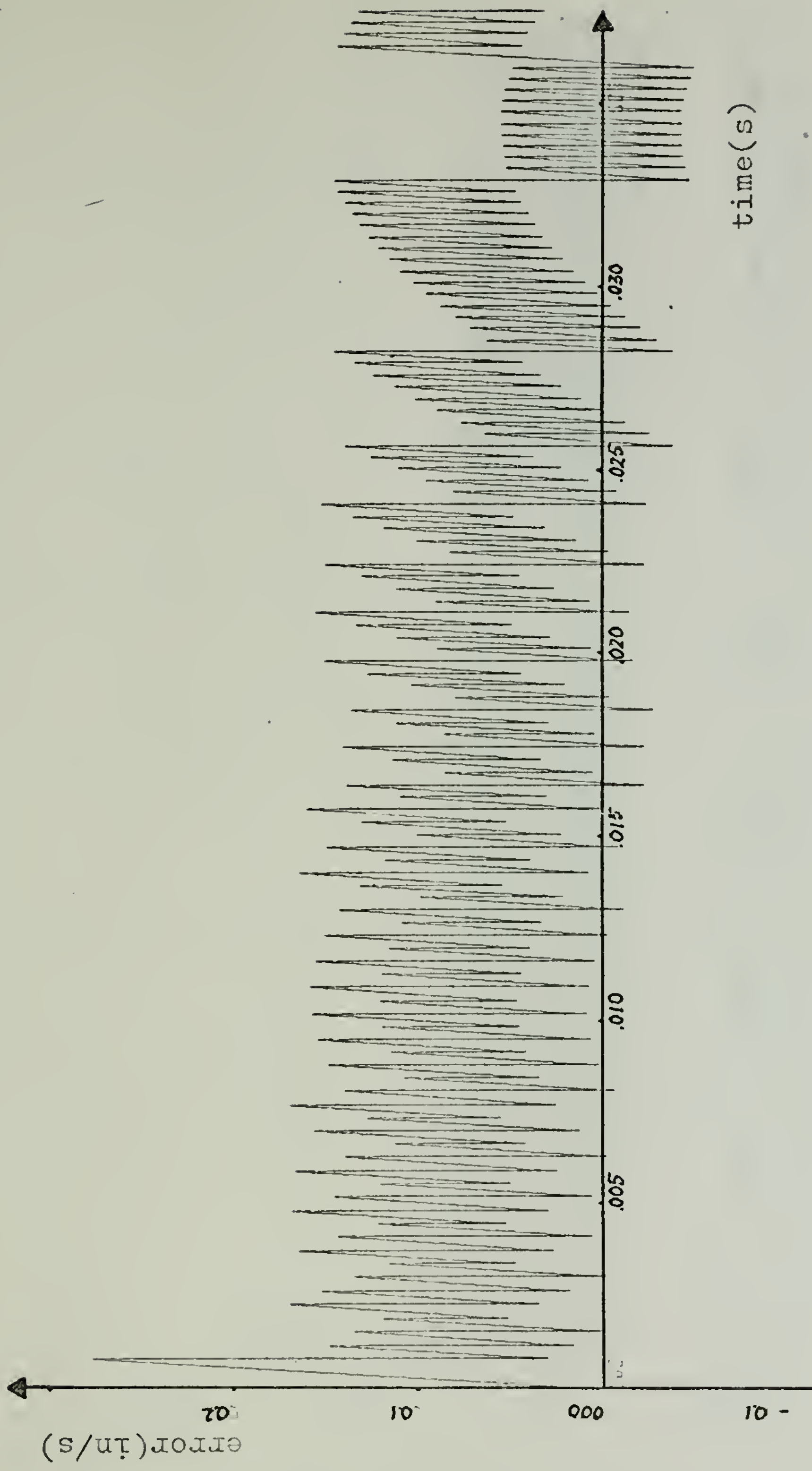


Figure 15.
Total error vs time,velocity=320*t
acceleration error plus truncation error

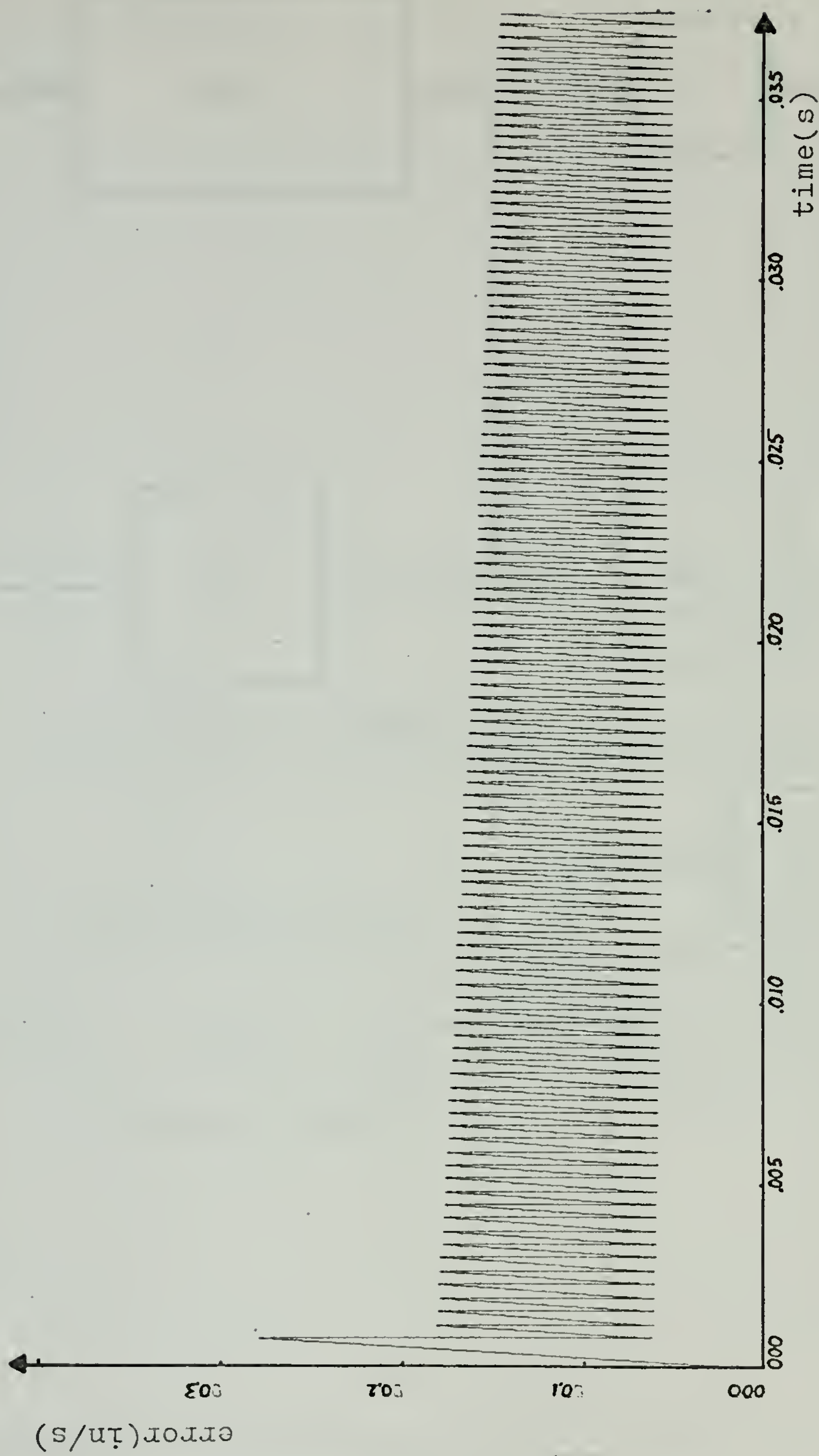


Figure 16.
Total error vs time,velocity=320*t
acceleration error only

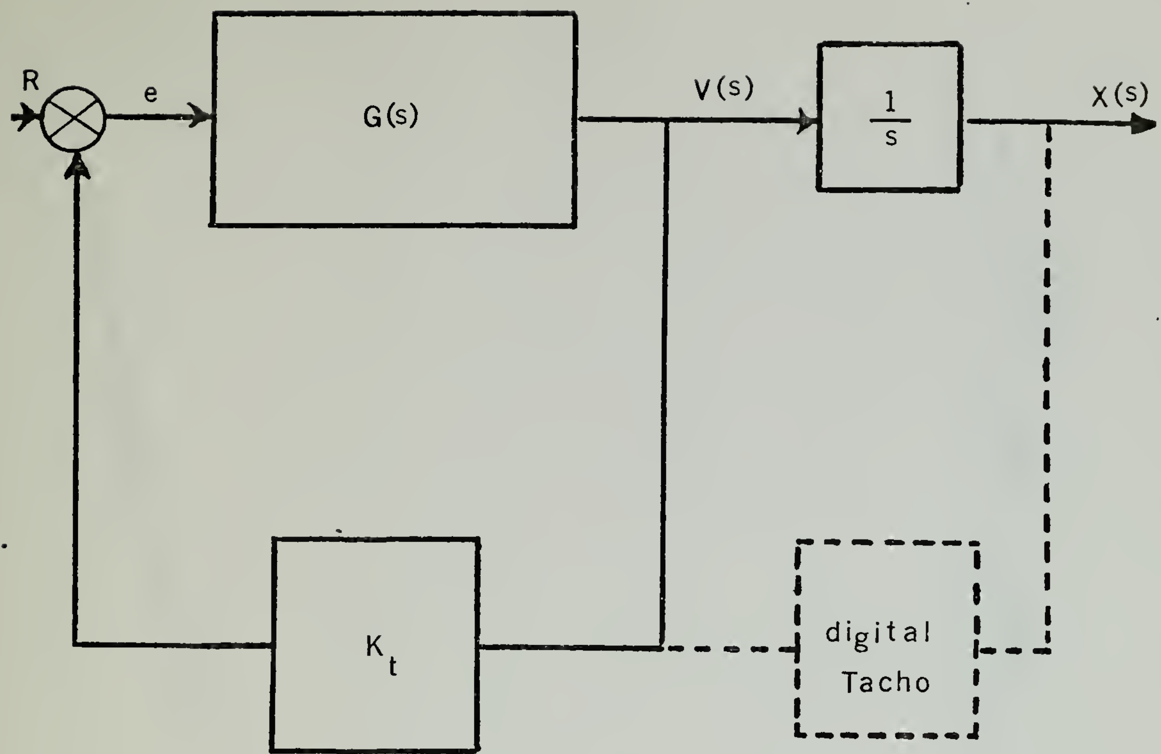


Figure 17.
Block diagram of first order velocity control

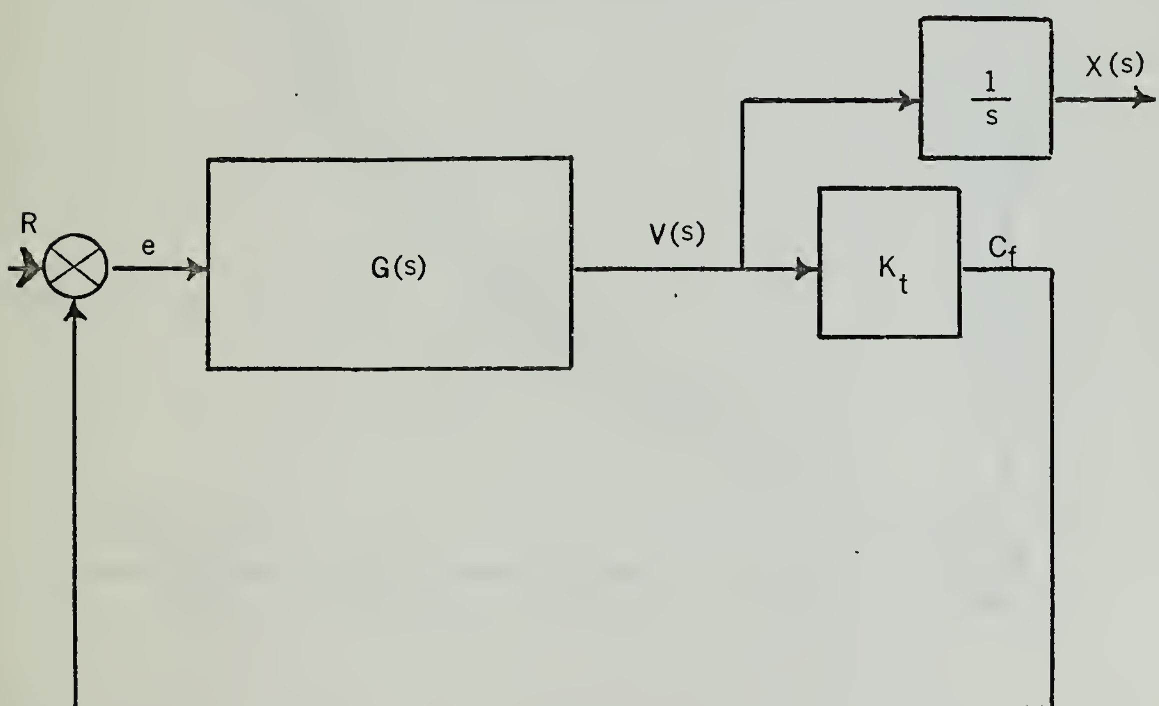


Figure 18.
Block diagram for error analysis

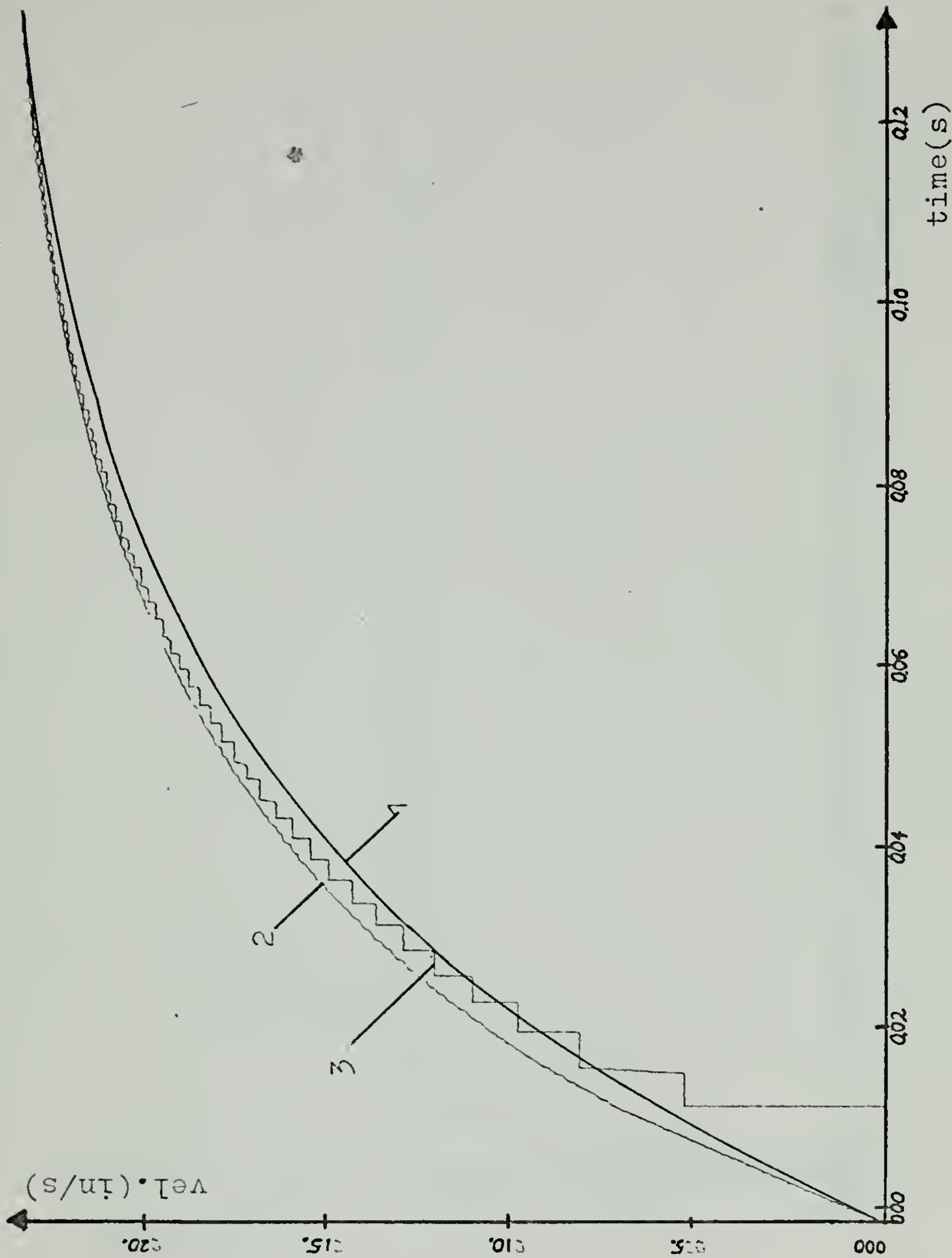


Figure 19.
1st order system, ppr=167 input=25
analog=1, digital=2, measured=3

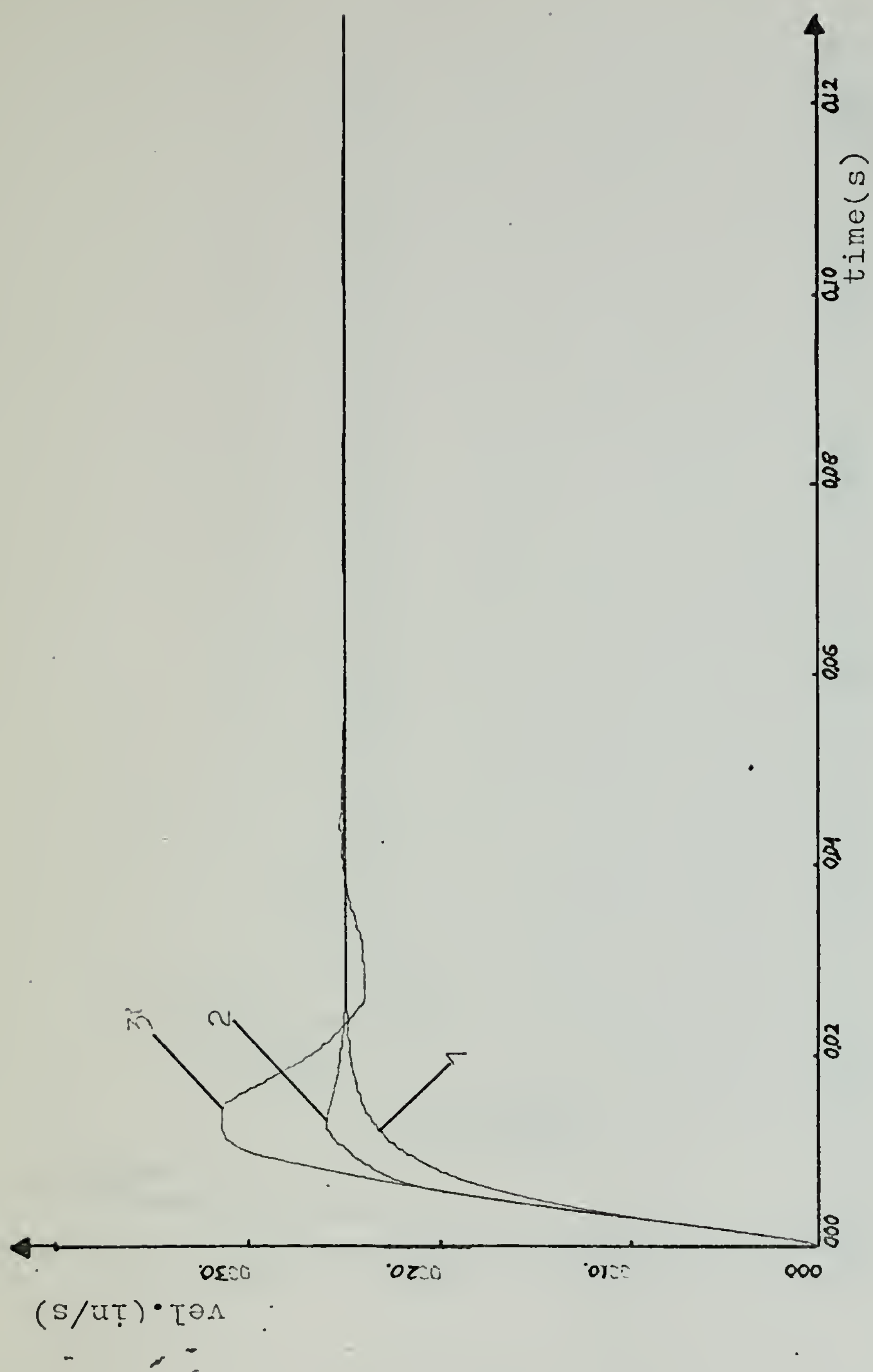


Figure 20.
2nd order system, ppr=42, delt=0.006
analog=1,sampled=2, digital=3, input=25

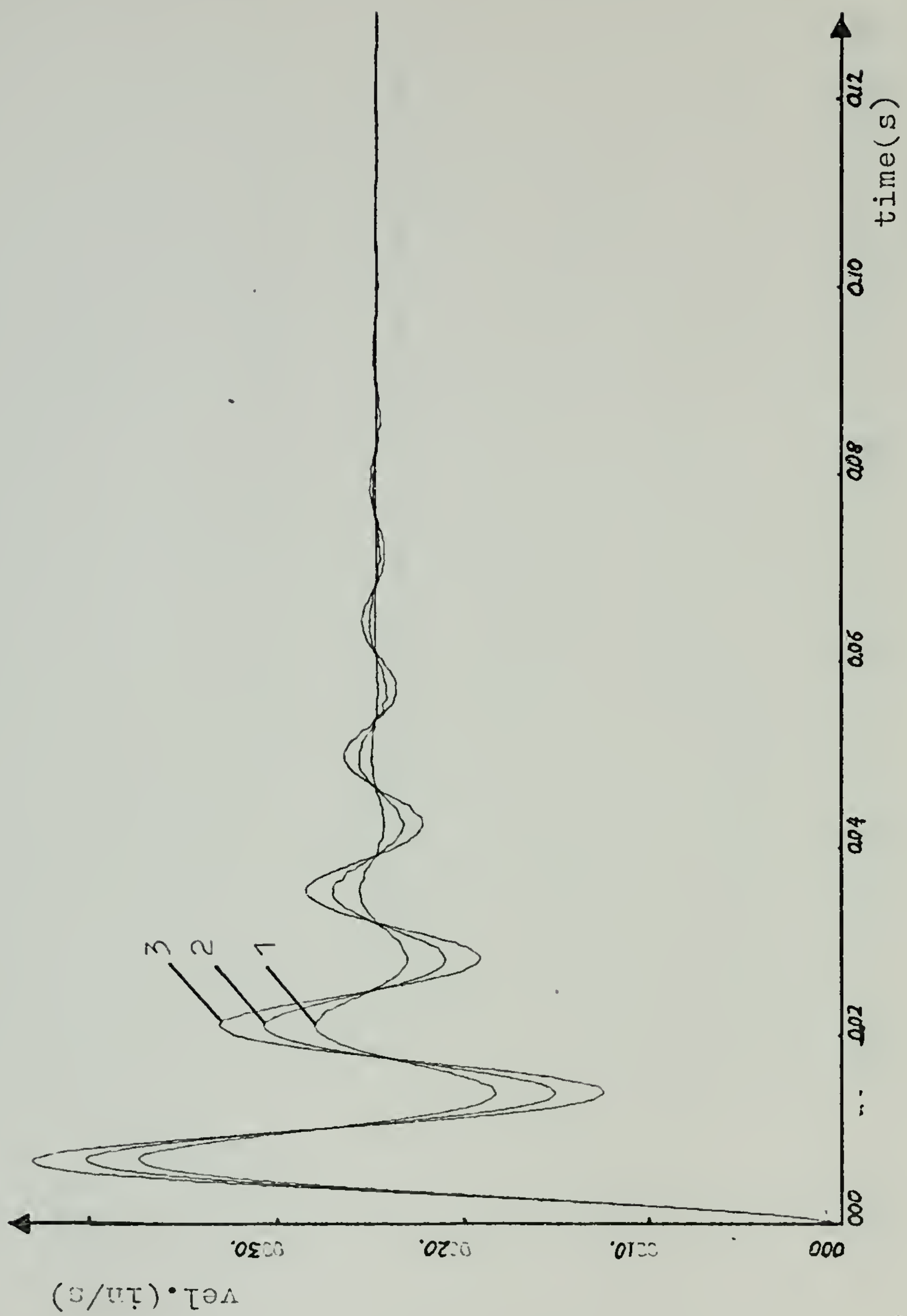


Figure 21.
Quadratic 2nd order system, ppr=333, delt=0.001
analog=1, sampled=2, digital=3, input=25

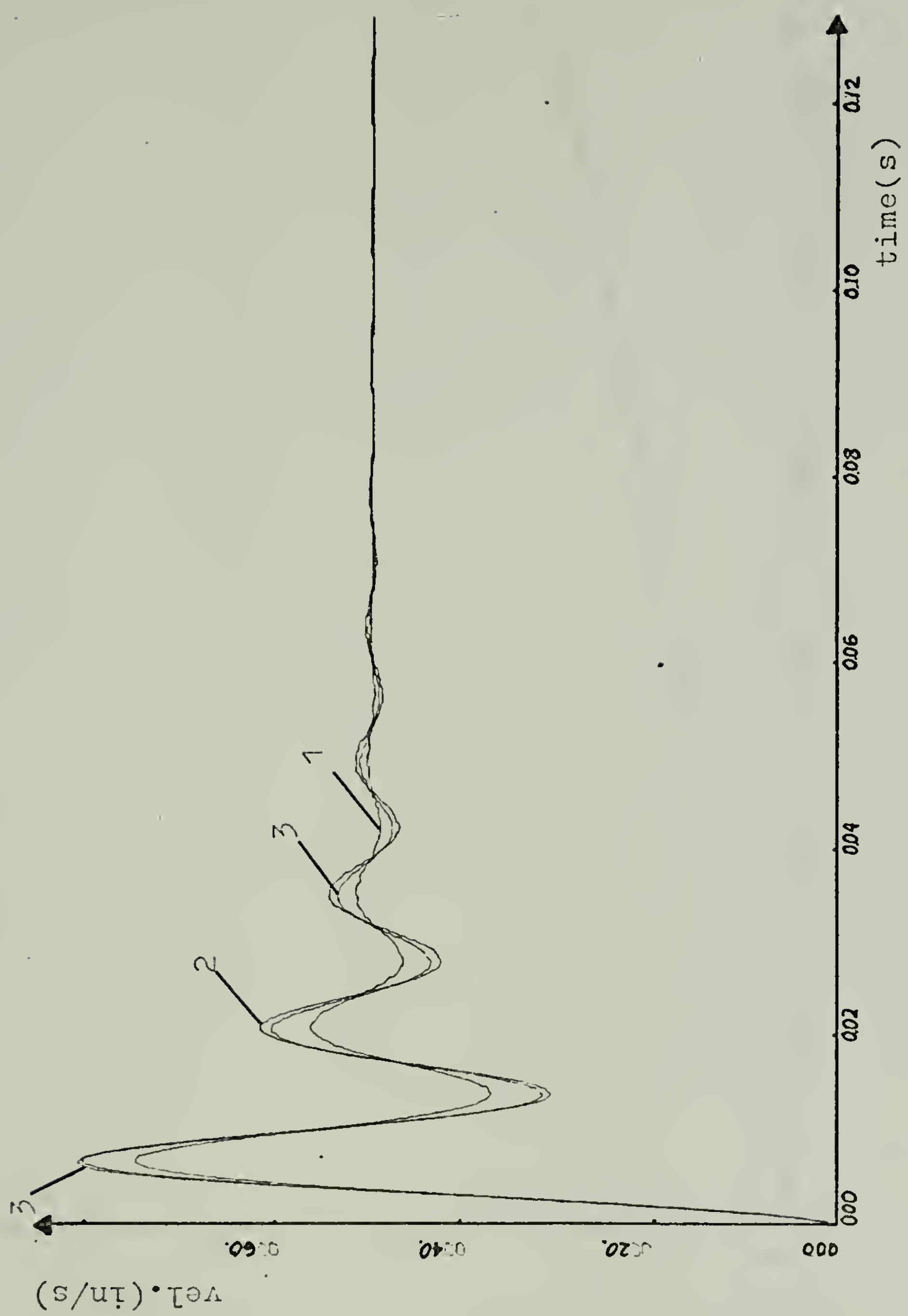


Figure 22.
Quadratic 2nd order system, ppr=333, delt=0.001
analog=1, sampled=2, digital=3, input=50

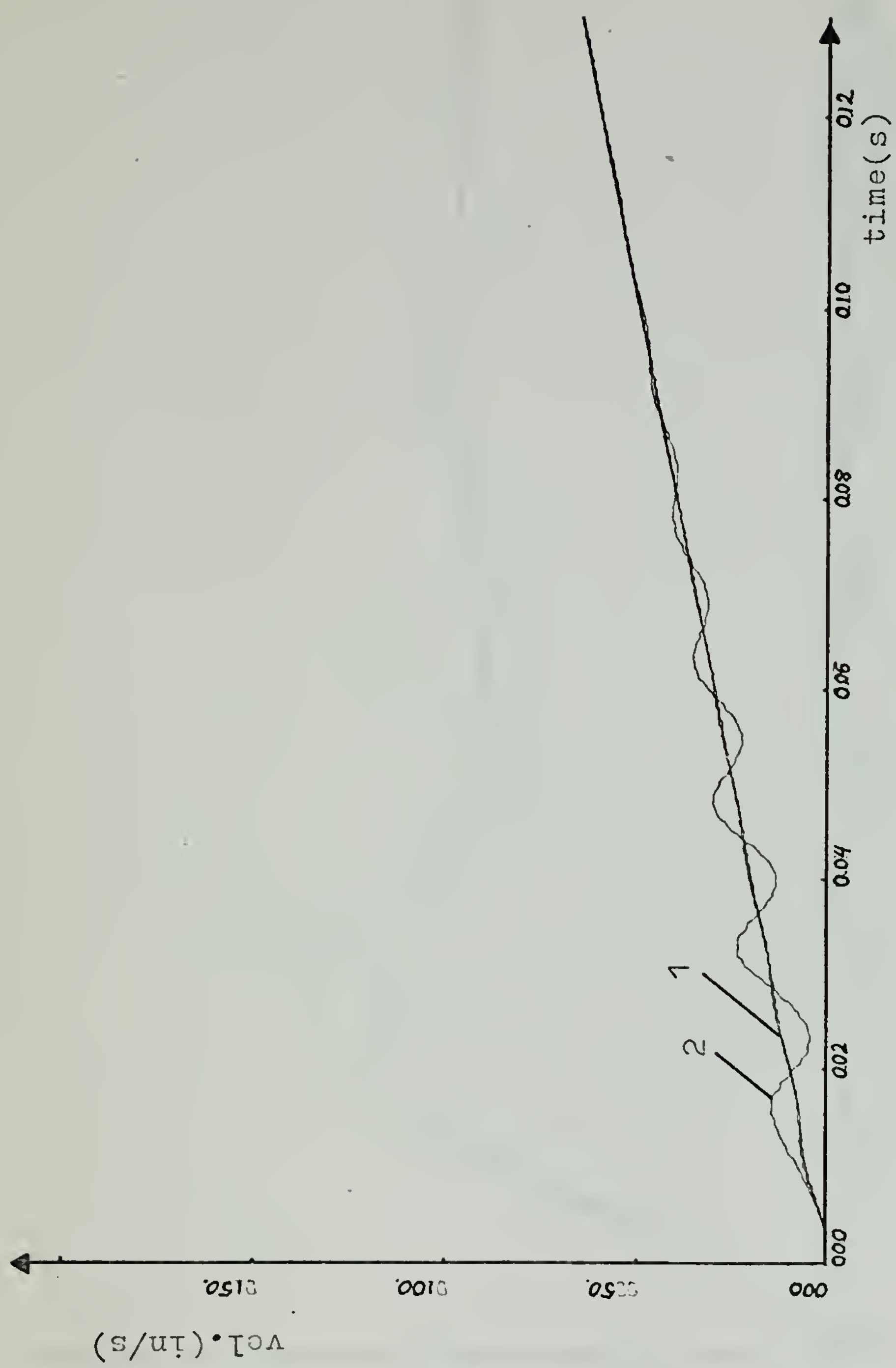


Figure 23.
Quadratic 2nd order system, ppr=167, delt=0.003
analog and sampled=1,digital=2, ramp=500t

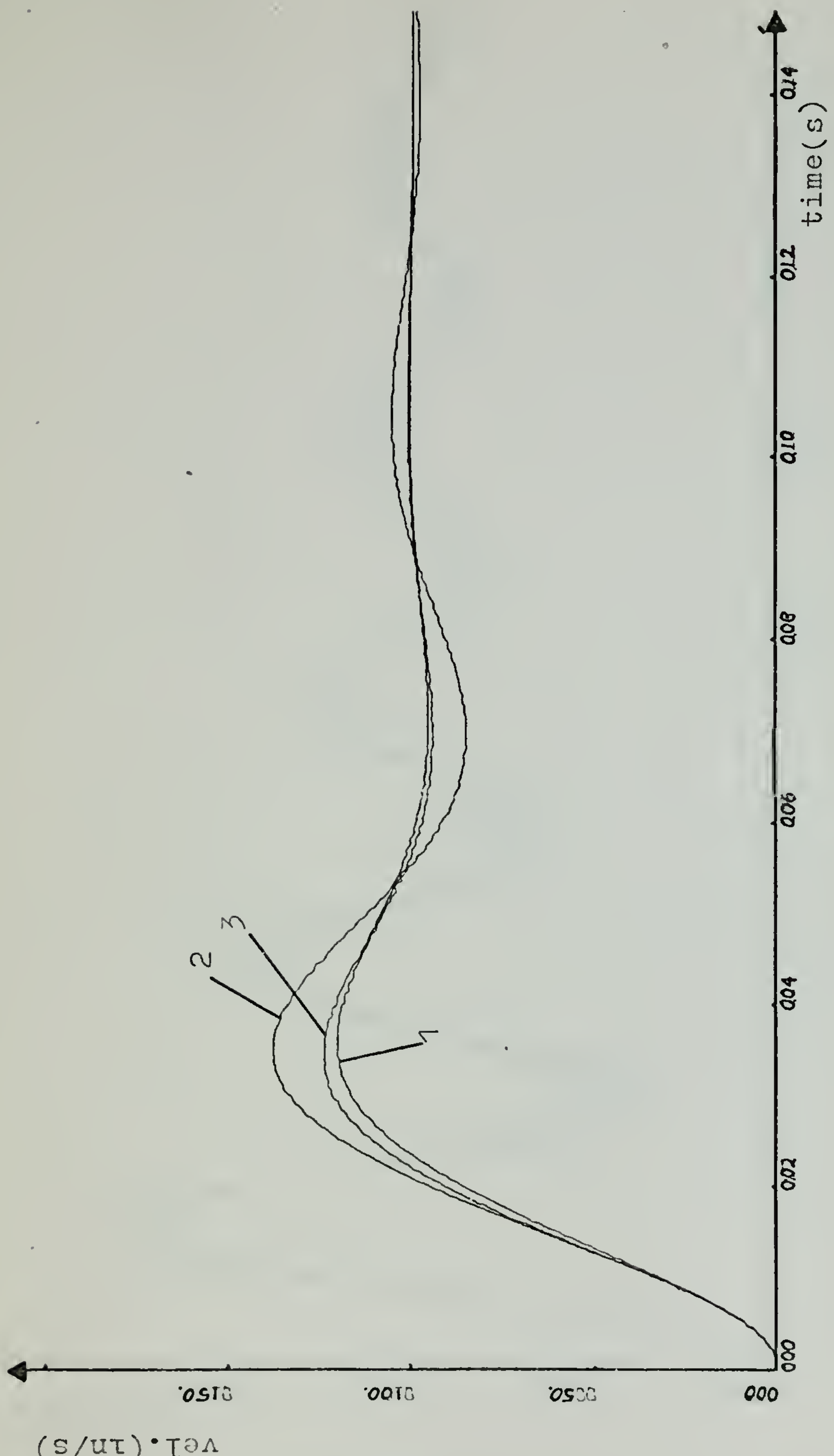


Figure 24.
3rd order damped system, ppr=167, delt=0.006
analog=1, sampled=2, digital=3, input=100

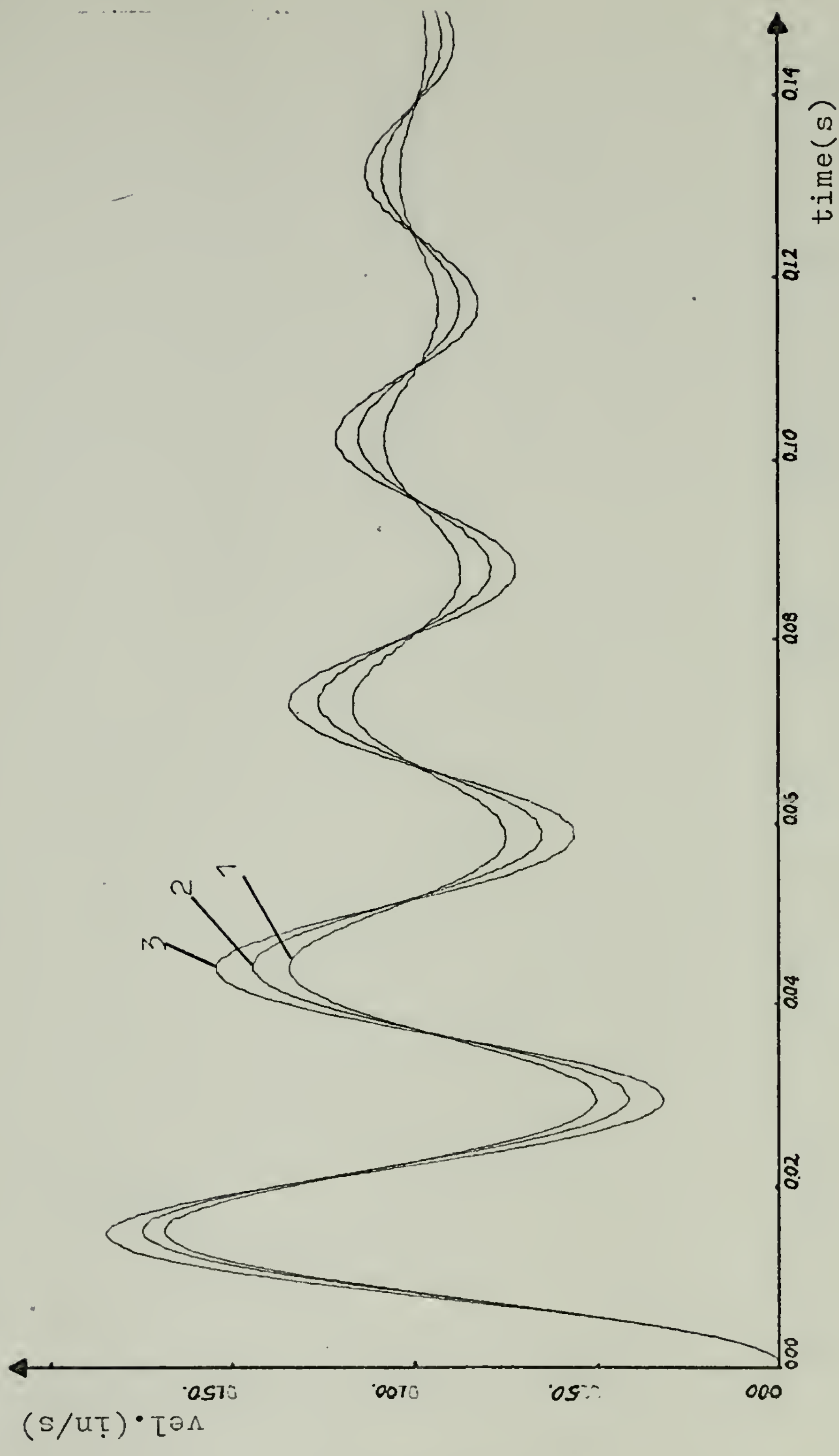


Figure 25.
3rd order underdamped, ppr=167, delt=0.0005
analog=1, sampled=2, digital=3, input=100

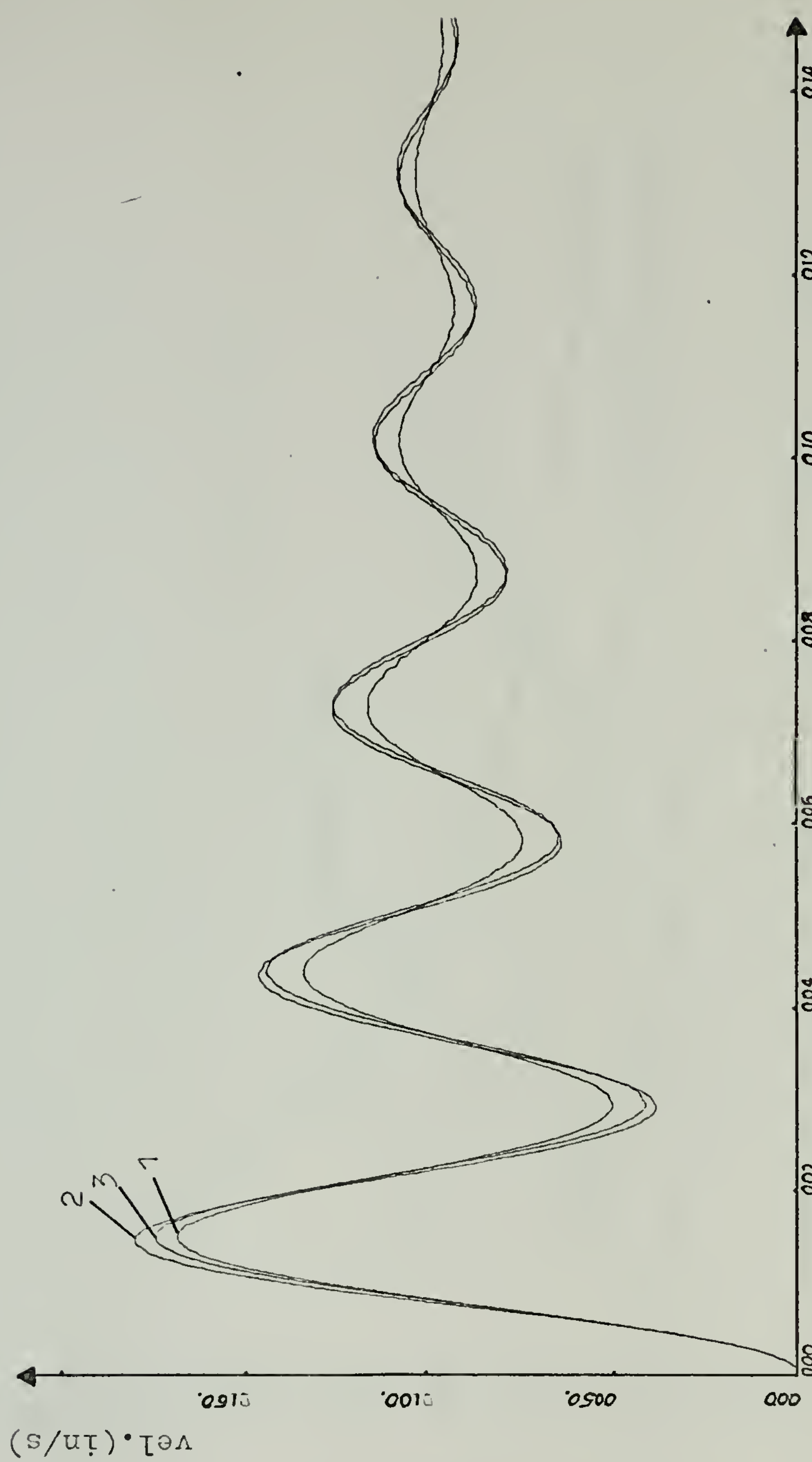


Figure 26.
3rd order underdamped, with first order hold
analog=1, sampled=2, digital=3, input=100

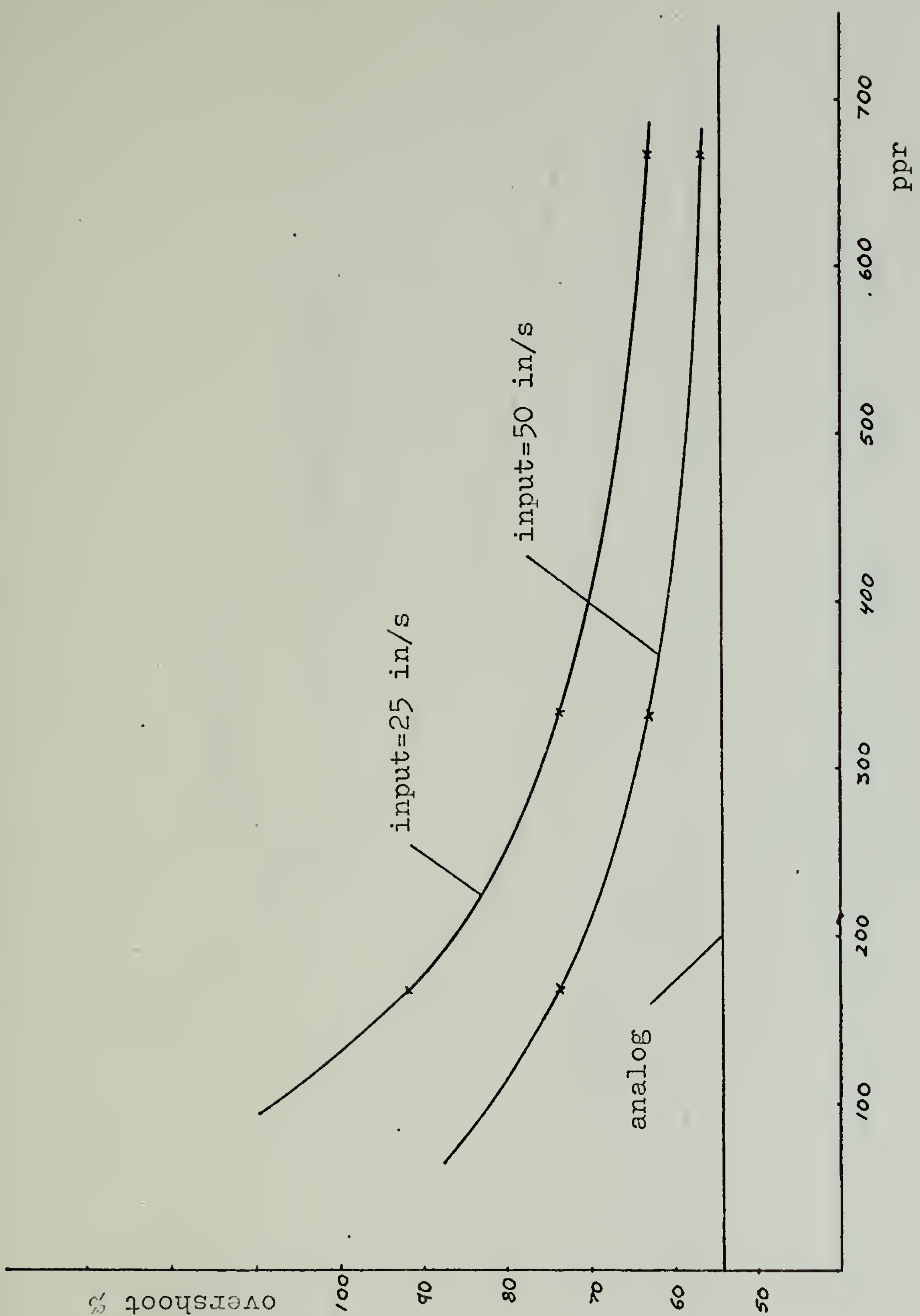


Figure 27.
2nd order quadratic system, input=25 and 50
amplitude vs ppr compared to analog response

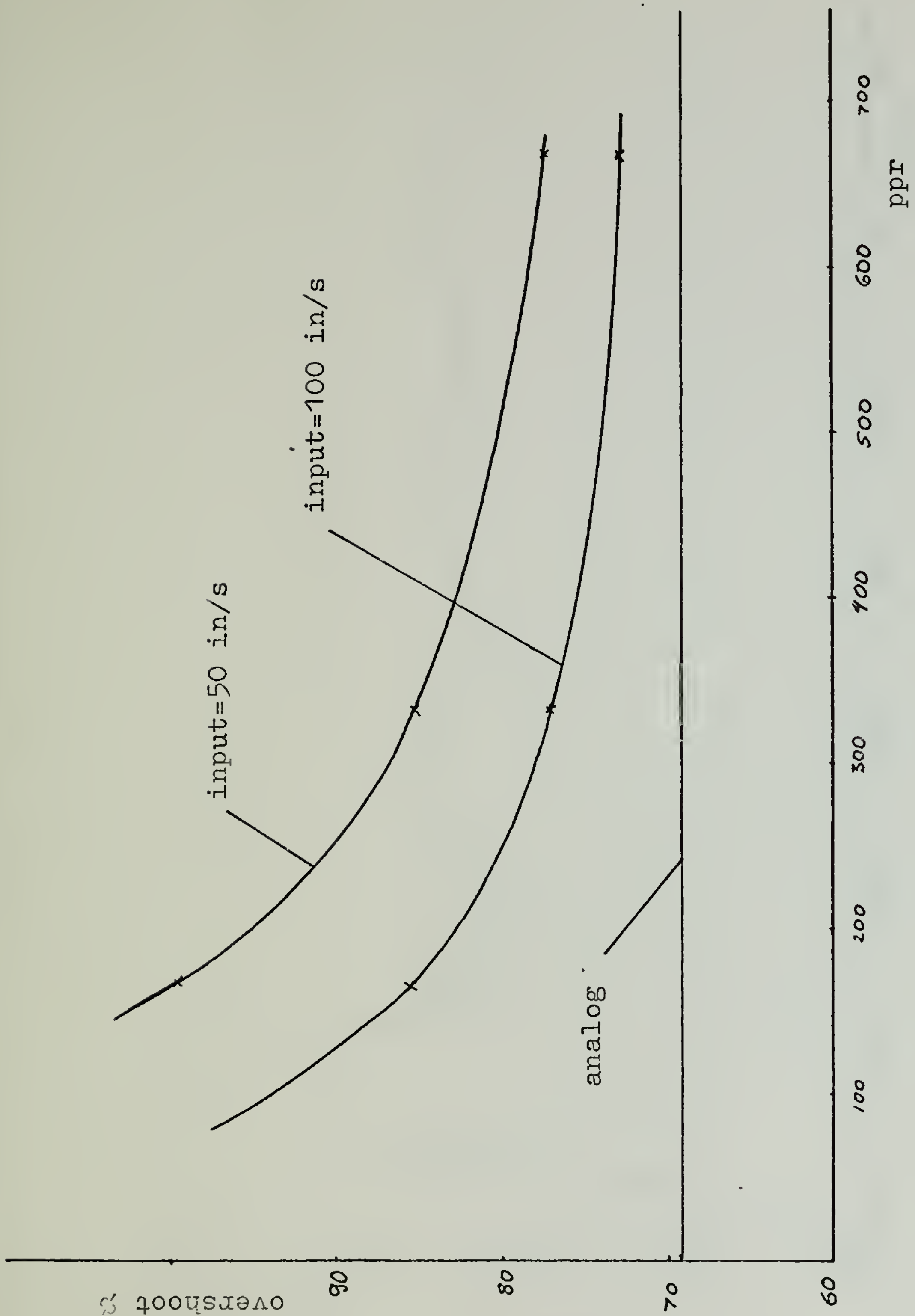


Figure 28.
3rd order underdamped system, input=50 and 100
amplitude vs ppr compared to analog response

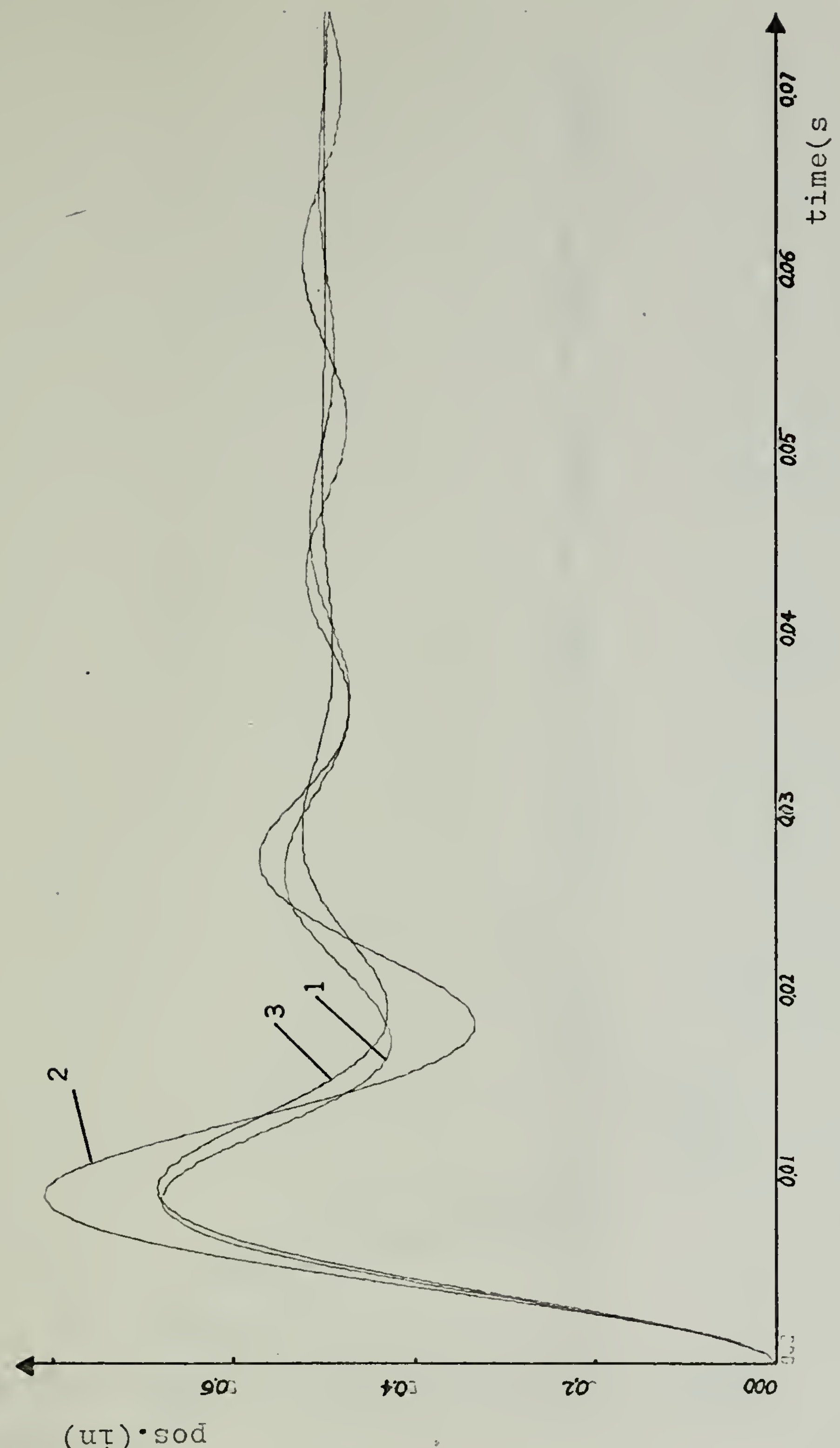


Figure 29.
2nd order position system, ppr=83, delt=0.01
analog=1, sampled=2, digital=3, input=0.5

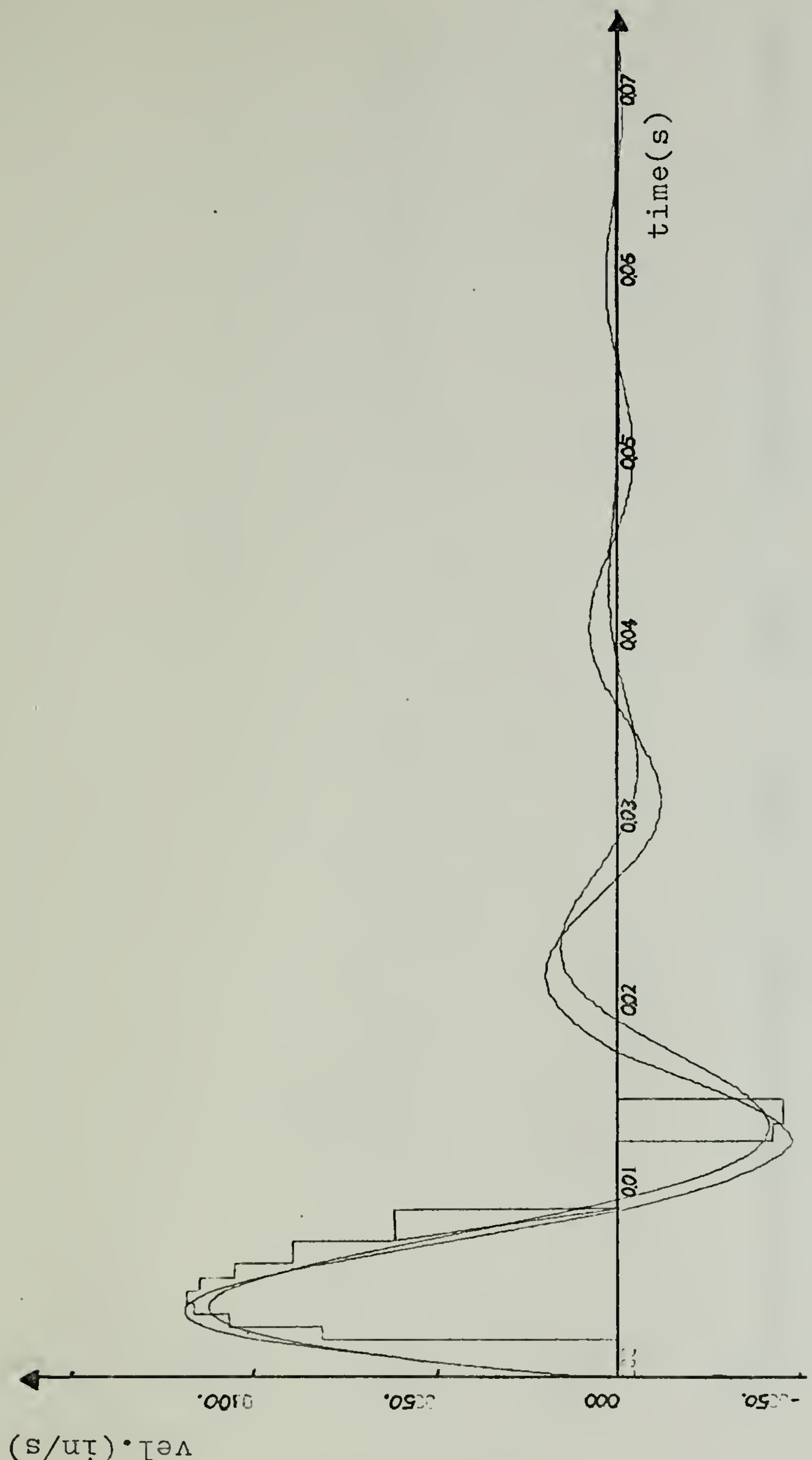


Figure 30.
2nd order position system, velocity vs time
actual and measured velocity vs analog

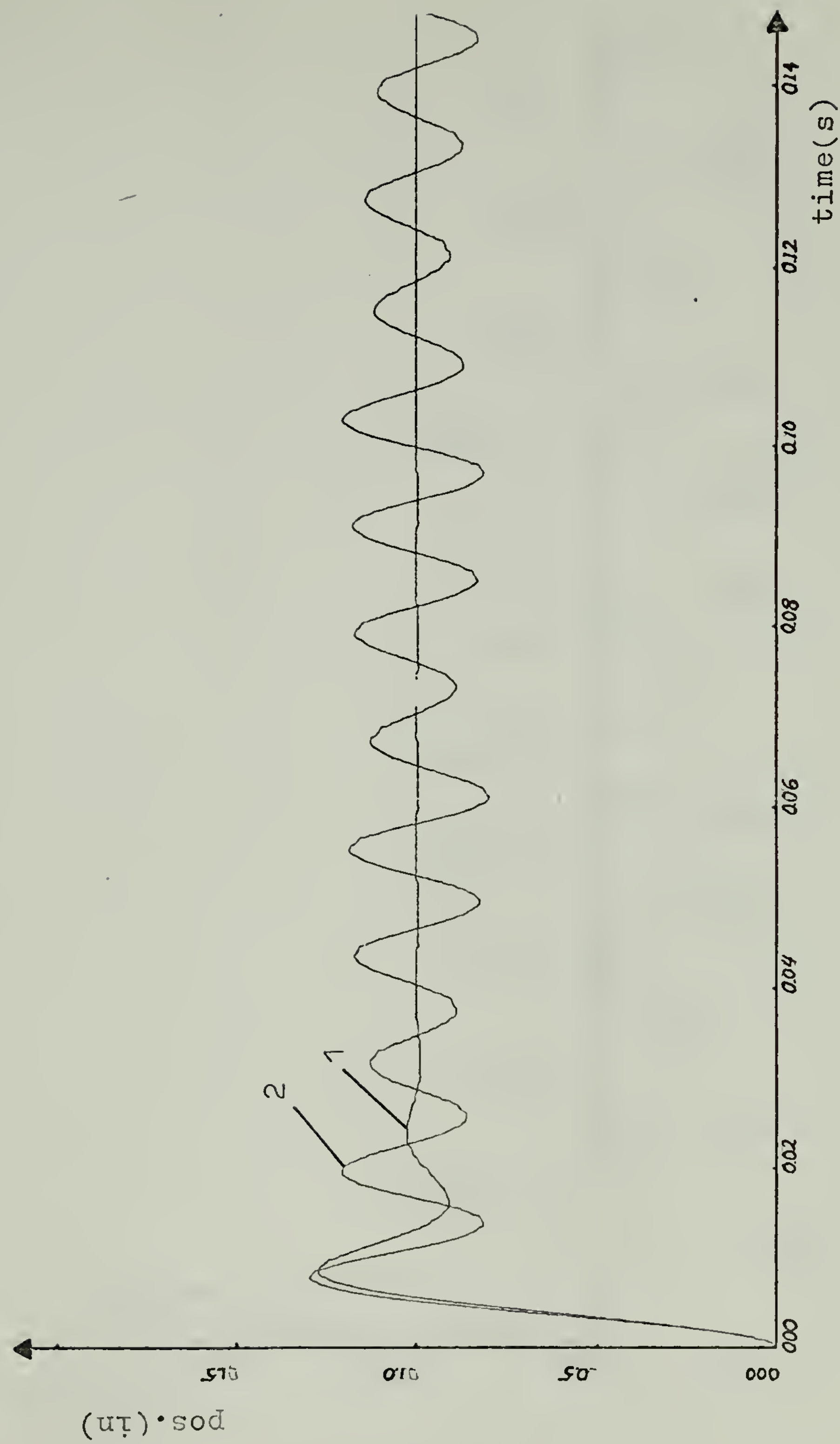


Figure 31.
3rd order position system, ppr=83
analog=1, digital=2, input=1.0

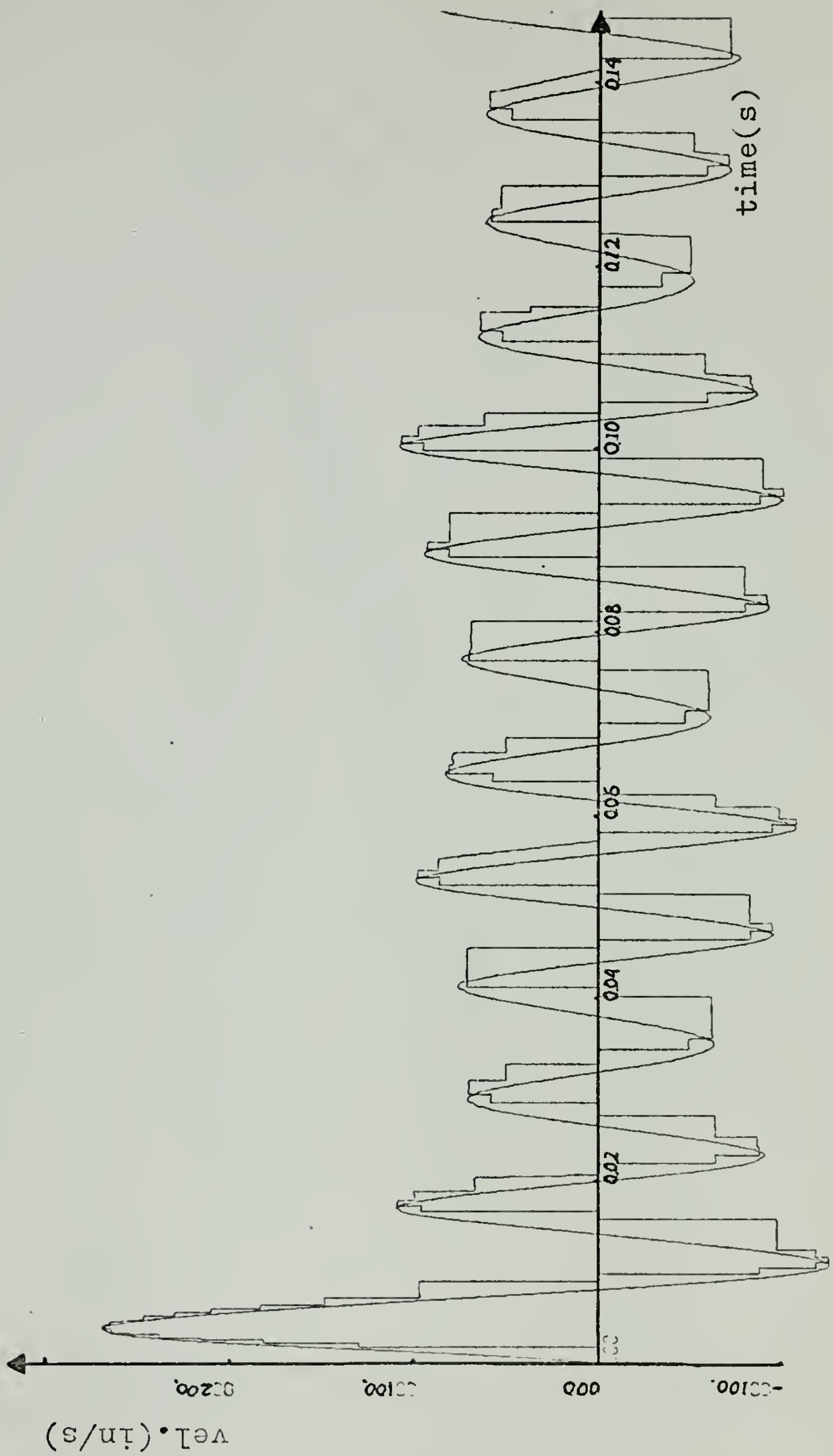


Figure 32.
3rd order position system, velocity vs time
actual and measured velocity

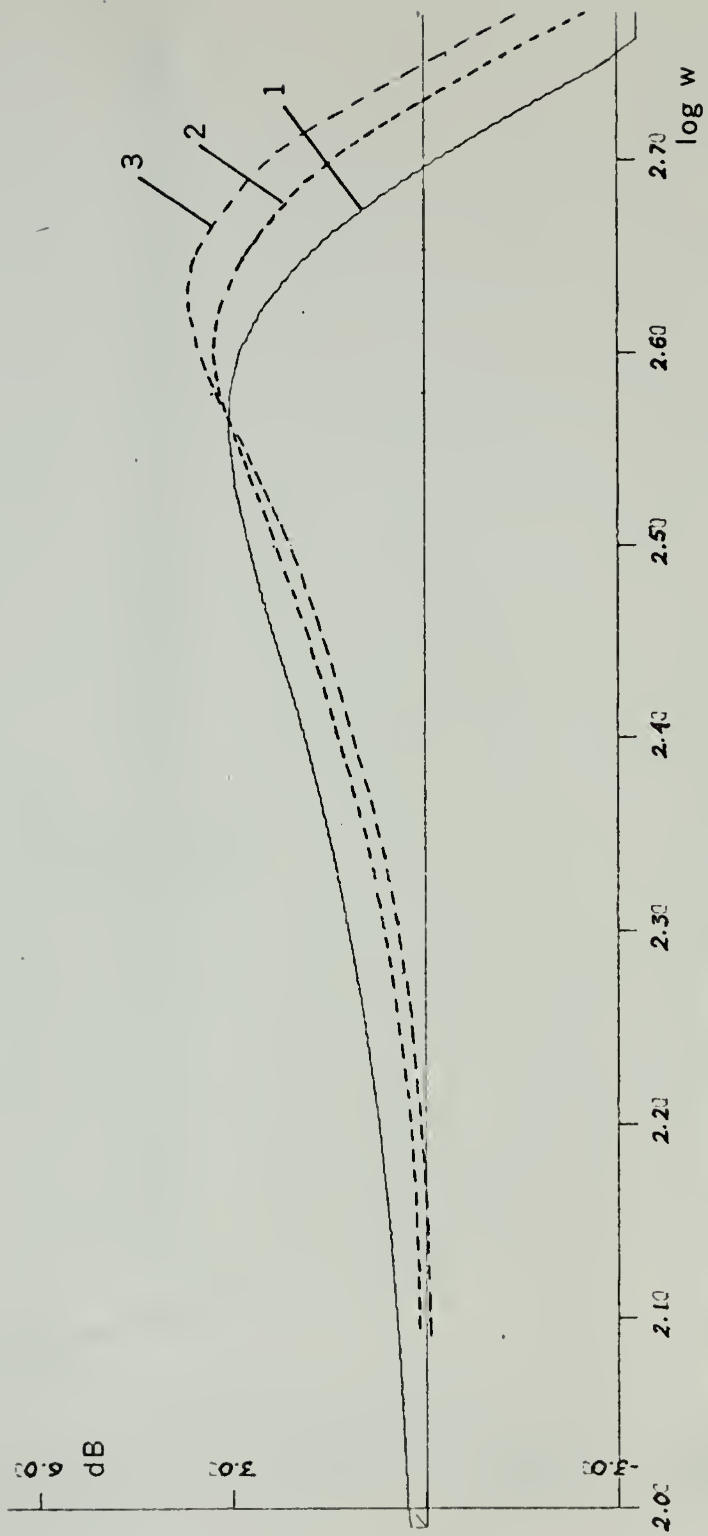


Figure 33.
Frequency response of analog and digital system
analog=1, digital with ppr83=2 and ppr42=3

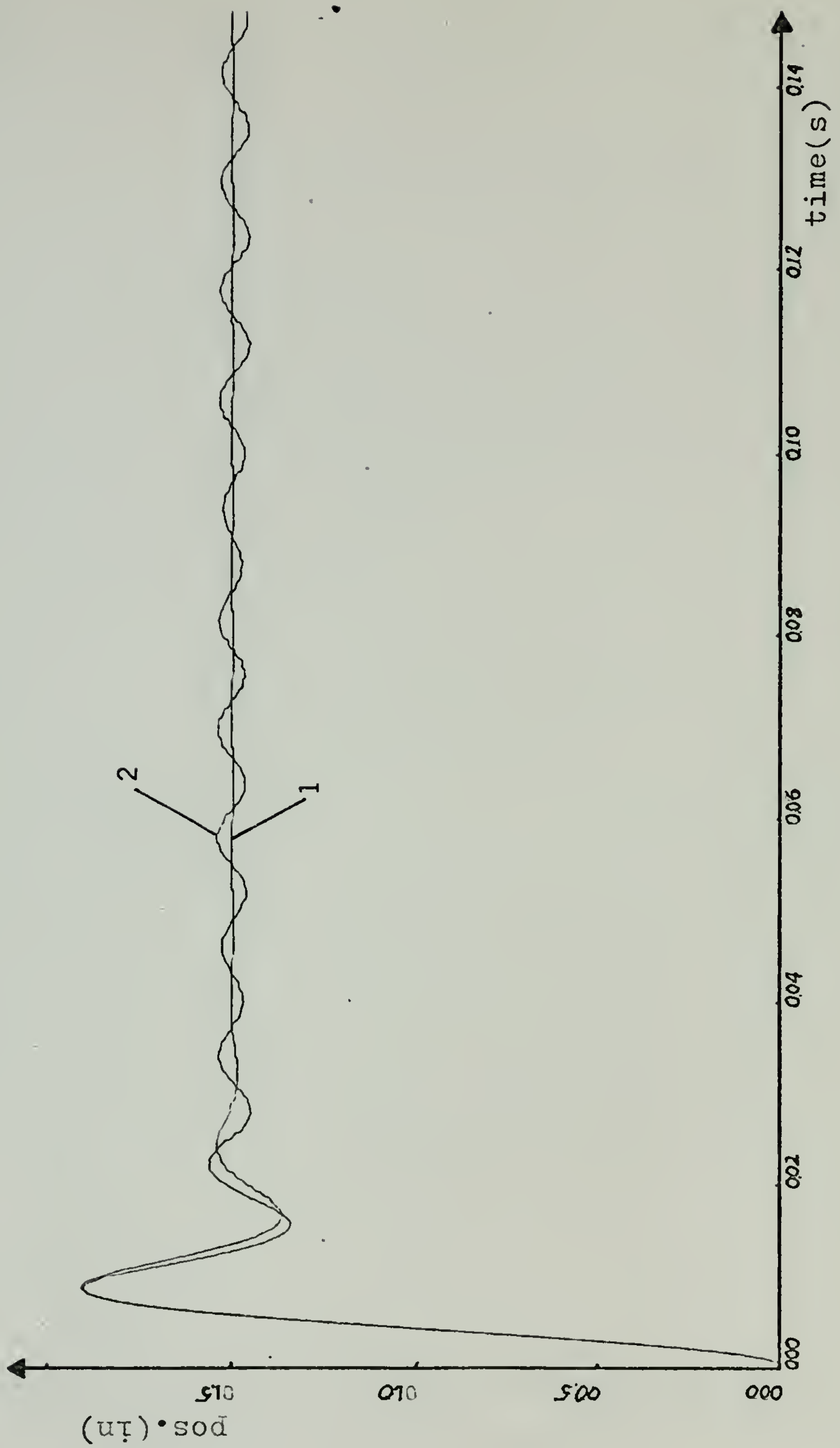


Figure 34.
3rd order position system, ppr=333, input=1.5
analog=1, digital=2, zero order hold

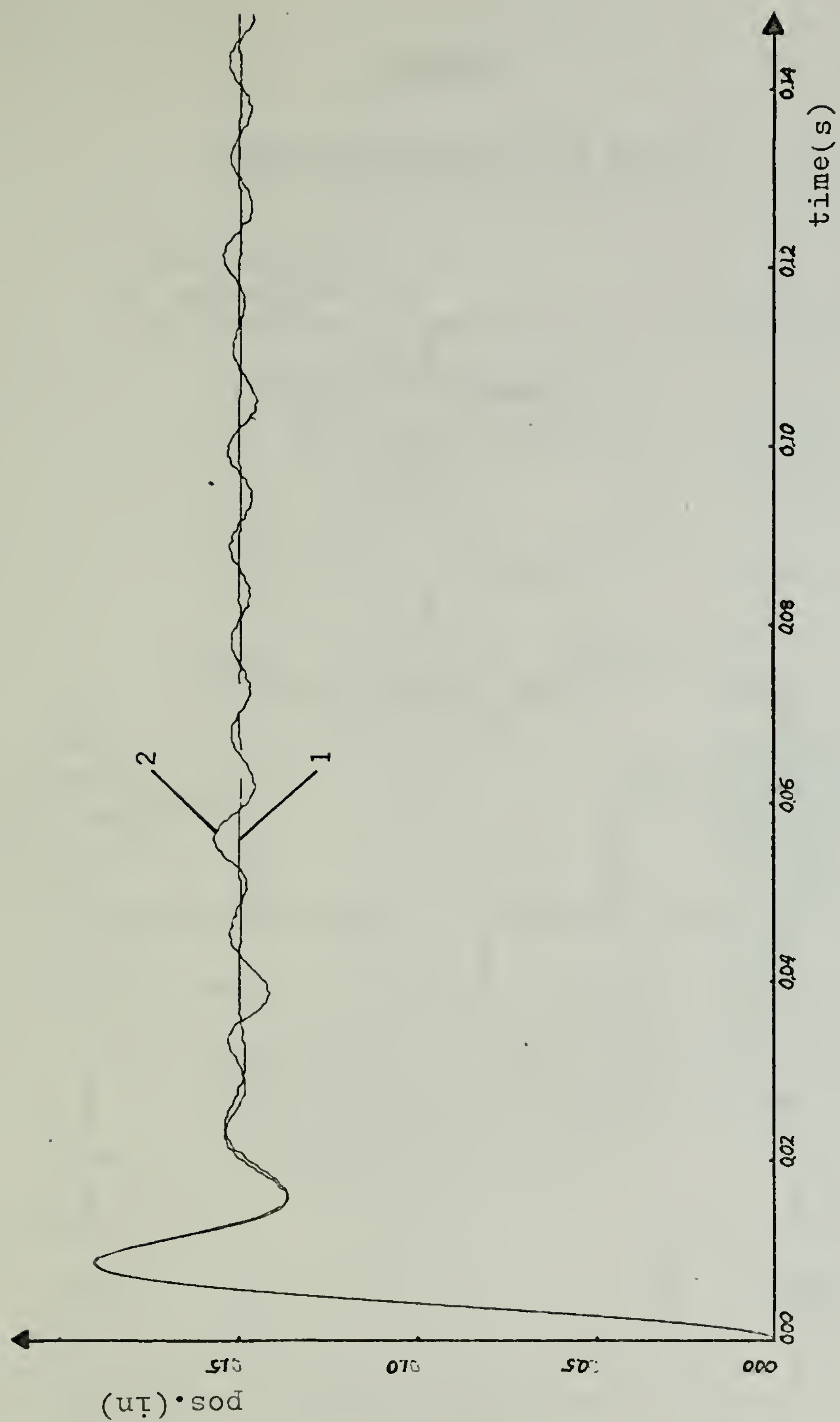
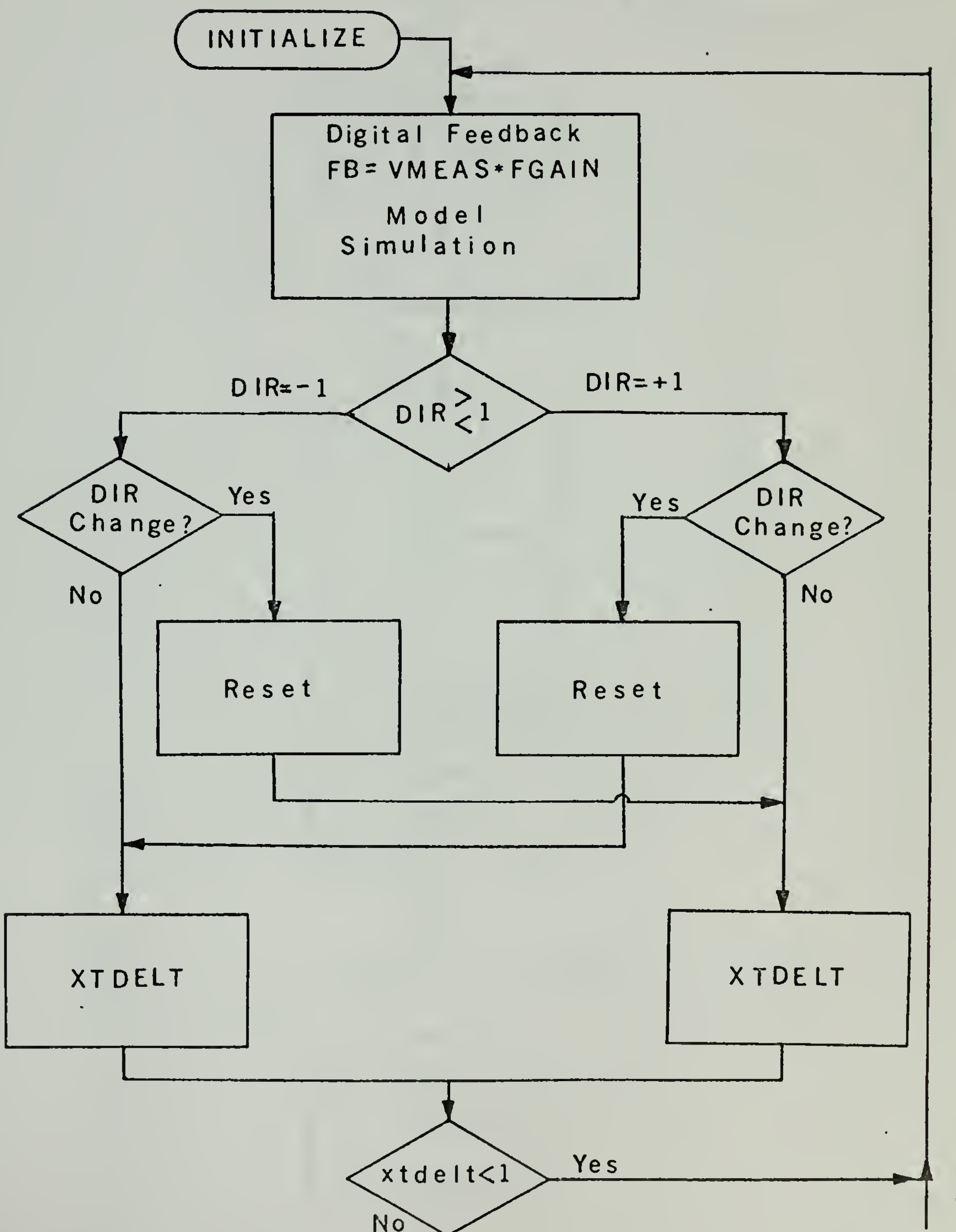
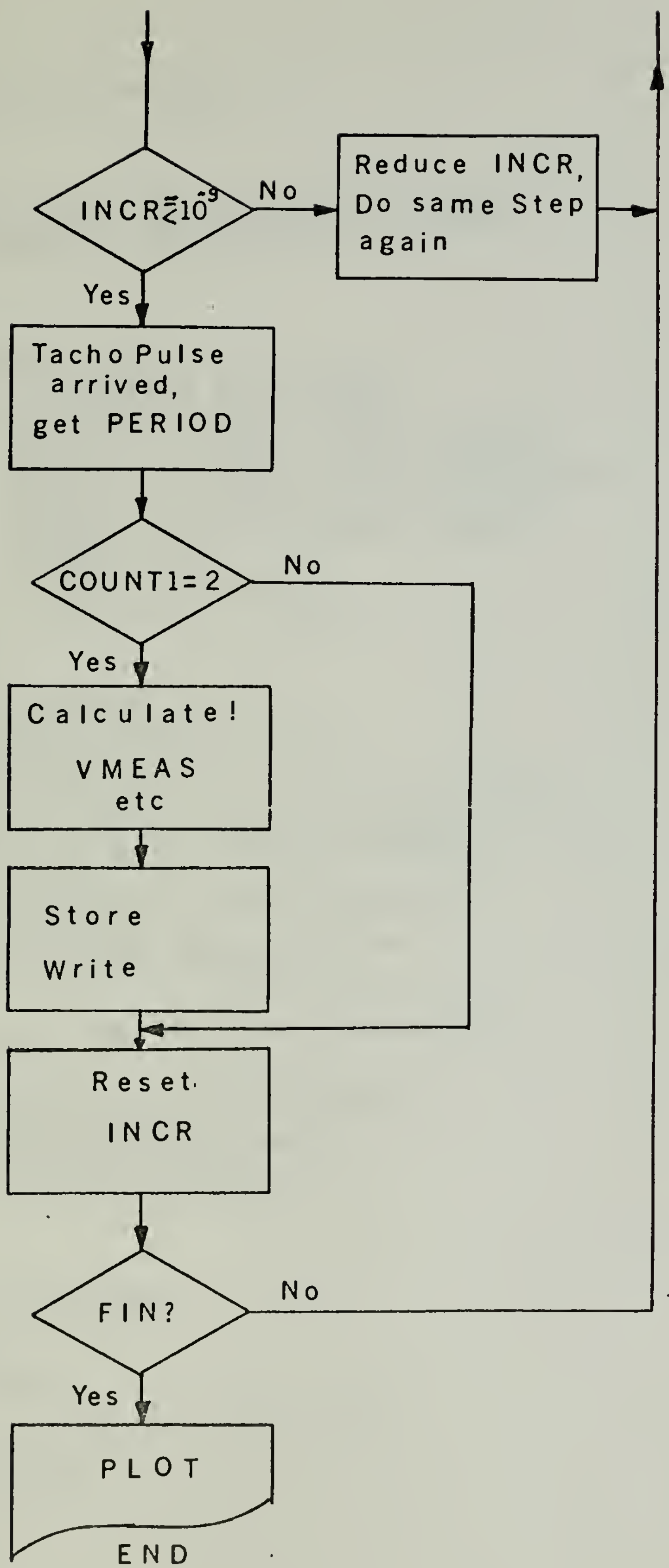


Figure 35.
3rd order position system, ppr=333, input=1.5
analog=1, digital=2, first order hold

APPENDIX

TACHOMETER MODEL FLOW CHART




INCR ≷ 10^{-9}
No
Reduce INCR, Do same Step again
Yes
Tacho Pulse arrived, get PERIOD
COUNT1=2
No
Yes
Calculate!
VMEAS etc
Store
Write
Reset INCR
FIN?
No
Yes
PLOT
END

COMPUTER PROGRAM

```
C
C           FORTRAN SIMULATION OF DIGITAL TACHOMETER
C           ANALOG,DIGITAL AND SAMPLER FEEDBACK
C           SAMPLE PROGRAM INCLUDED
C
C
C
C
C           TACHO PARAMETERS
C           RT        TACHOMETER DISC RADIUS
C           TACRES    TACHOMETER RESOLUTION
C           KTACHO    NUMBER OF TACHO PULSES PER INCH
C           DR        DISTANCE BETWEEN TACHO PULSES
C           TACRV     NUMBER OF TACHO PULSES PER RADIANS
C           TACSH     ERROR IN INITIAL TACHO POSITION
C           NCLOCK    NUMBER OF CLOCKPULSES COUNTED
C           COUNT     ACTUAL COUNT OF TACHO MARKERS
C           LCOUNT    LAST COUNT
C           RCOUNT    SECOND LAST COUNT
C           DIR       DIRECTION OF VELOCITY
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      EQUIVALENCE (TITLE,RTB(5))
      REAL*8 LCTIME,INCR,KTACHO,TITLE(12)
      REAL    *4 RTB(28)/28*0.0/,S
      INTEGER*4 ITB(12)/12*0/,PTS,SWITCH,COUNT,COUNT1
      INTEGER*4 RCOUNT,CONECT
      DIMENSION TA(600),TD(600),XA(600),XD(600),VA(600)
      DIMENSION TS(600),XS(600),VS(600),VD(600)
      DIMENSION Y(10),F(10)
      DATA ITB(2),ITB(3),ITB(4),ITB(5)/0,9,4,1/
      DATA ITB(6),ITB(8)/1,1/
      DATA TACSH,RT,LCTIME/0.75D0,1.D0,0.D0/
      DATA POLE1,POLE2/100.D0,1000.D0/
C
C
C           INITIALIZE VARIABLES AND PARAMETERS
C
C
      READ(5,102) GAIN,FGAIN,VBIAS,OSCF,DELT
    1 READ(5,101,END=999)TITLE
      READ(5,102) TACRES,DELAY,FIN,DPLOT
      READ(5,102) TIME,V2,X2,VMEAS
      READ(5,103)RTB(1),CONECT,SWITCH
      J=1
      K=1
      N=1
      NT=0
      TACRV=TACRES/6.283185308
      KTACHO=TACRV/RT
      DR=1.D0/KTACHO
      W=6.28315D0*FREQ
      XBIAS=VBIAS*TIME
      DIR=1.D0
      CONCNT=(50.D0+X2+XBIAS)*KTACHO+TACSH
      UCOUNT=0.D0
      COUNT=IDINT(CONCNT)
      LCOUNT=COUNT
      RCOUNT=10
      PERIOD=0.0D0
      LCTIME=0.0D0
      ERROR1=0.0D0
      VDOT=0.D0
```

```
      VOLD=0.D0
      VSAMPL=0.D0
      Y(1)=X2
      Y(2)=V2
      Y(3)=0.0D0
      Y(4)=0.0D0
      Y(5)=0.D0
      Y(6)=0.D0
      INCR=DELT
      XA(1)=X2
      XD(1)=X2
      VA(1)=V2
      VD(1)=V2
      TA(1)=TIME
      TD(1)=TIME
      TS(1)=TIME
      XS(1)=X2
      VS(1)=V2
      TNET2=0.0D0
      TIMED=TIME
      DNEXT=TIME+DPLOT
      PLOTT=FIN/500.D0
      PLOTD=PLOTT
      VELCOM=0.0
      TACSH=0.001
C
C
      WRITE(6,100)
      WRITE(6,110)
      WRITE(6,120)GAIN,FGAIN,KTACHO
      WRITE(6,130)
      WRITE(6,140)
      WRITE(6,150)V2,X2,VMEAS,FBTIME
      GO TO(5,6,7),CONECT
    5 WRITE(6,106)
      GO TO 12
    6 WRITE(6,107)
      GO TO 12
    7 WRITE(6,109)
   12 WRITE(6,104)TIME,INCR
      WRITE(6,105)FIN,DPLOT
      WRITE(6,180)
      WRITE(6,190)
      WRITE(6,200)TIME,PERIOD,TNET2,V2,VMEAS,X2,ERROR1,COUNT
      NPRINT=1
C
C
      DO 90 I=1,1000000
      GO TO (40,57,58),CONECT
C
C
C              DIGITAL TACHO FEEDBACK
C
C
   57 FB=VMEAS*FGAIN
C
C              STORE ALL PRESENT VALUES FOR FUTURE REFERENCE
C
      TOLD=TIME
      V2OLD=V2
      X2OLD=X2
      Y1OLD=Y(1)
      Y2OLD=Y(2)
      Y3OLD=Y(3)
      Y4OLD=Y(4)
      Y5OLD=Y(5)
      Y6OLD=Y(6)
      VMOLD=VMEAS
      CONCNL=CONCNT
      GO TO 40
C
C
```

```
C                 SAMPLE AND HOLD FEEDBACK
C
C
   58 IF((TIME-TIMED).LT.DELAY)GO TO 59
      VSAMPL=V2
      TIMED=TIME
   59 FB=VSAMPL*FGAIN
C
C
C                 SECOND ORDER VELOCITY CONTROL,QUADRATIC
C
C
   40 VELCOM=100.D0
      POLE1=189.7D0
      IF(CONECT.EQ.1)FB=Y(2)*FGAIN
      TNET2=(VELCOM-FB-Y(2))*GAIN
      F(1)=Y(2)
      F(2)=Y(3)
      F(3)=TNET2-POLE1*Y(3)
      S=RKLDEQ(3,Y,F,TIME,INCR,NT)
      X2=Y(1)
      V2=Y(2)
      IF(S-1.00)70,40,300
C     ERROR STOP
   70 WRITE(6,250)
      STOP
C
C
  300 IF((CONECT.EQ.1).OR.(CONECT.EQ.3))GO TO 301
C
C
C                 DIGITAL COUNT SIMULATION
C
C
C                 CONCNT IS INTEGRATED UNTIL IT INDICATES THAT
C                 A TACHO PULSE HAS ARRIVED.COUNT IS THEN IN-
C                 CREASED AND THE NUMBER OF COUNTED OSCILLATIONS
C                 DETERMINED.CONCNT IS INITIALLY SET TO A LARGE
C                 NUMBER TO PREVENT IT FROM COUNTING INTO
C                 NEGATIVE REGION DURING NEGATIVE VELOCITIES
C
C
      XBIAS=VBIAS*TIME
      CONCNT=(50.D0+X2+XBIAS)*KTACHO+TACSH
      IF(DIR)32,32,31
C
C                 CHECK FOR CHANGE IN DIRECTION,BY COMPARING
C                 THE CALCULATED DISTANCE WITH THE LAST VALUE
C
   31 IF(CONCNT-CONCNL)33,33,34
   32 IF(CONCNL-CONCNT)35,35,36
C
C                 CHANGE OF DIRECTION OCCURRED FROM + TO -
C
   33 UCCUNT=1.D0
      CCUNT=IDINT(CONCNT+UCCUNT)
      RCOUNT=LCOUNT
      LCOUNT=CCUNT
      VMEAS=0.D0
      DIR=-1.D0
      GO TO 36
C
C                 CHANGE OF DIRECTION OCCURRED FROM - TO +
C
   35 UCCUNT=0.D0
      CCUNT=IDINT(CONCNT)
      RCOUNT=LCOUNT
      LCOUNT=CCUNT
      VMEAS=0.D0
      DIR=1.D0
   34 XTDELT=CONCNT-COUNT
      GO TO 37
```

```
C
C
C                 TO SEARCH FOR THE CORRECT PULSE TIME,THE
C                 DISTANCE TRAVELED IS DETERMINED BY INTEGRATING
C                 IN DECREASING STEPS OF TIME.IF THE CALCULATED
C                 DISTANCE IS LARGER THAN THE DISTANCE BETWEEN
C                 TACHO PULSES,THE STEP SIZE IS REDUCED,THE
C                 PREVIOUS VALUES ARE RESTORED AND THE INTEGRA
C                 TION DONE AGAIN.WHEN THE STEP SIZE REACHES
C                 10D-8,THE RESULT IS TAKEN AS AN INDICATION
C                 OF A TACHOMETER PULSE,AND THE TIME IT TOOK
C                 FOR THE INTEGRATION IS THE PERIOD BETWEEN TWO
C                 TACHOMETER PULSES,THE SAMPLING PERIOD.
C
   36 XTDELT=COUNT-CONCNT
   37 IF(XTDELT.LT.1.D0) GO TO 301
      IF(INCR.LE.10.D-09) GO TO 38
C
C                 INTEGRATED TOO FAR ,RESET VALUES,GET NEW
C                 STEPSIZE AND INTEGRATE AGAIN
C
      TIME=TOLD
      X2=X2OLD
      V2=V2OLD
      Y(1)=Y1OLD
      Y(2)=Y2OLD
      Y(3)=Y3OLD
      Y(4)=Y4OLD
      Y(5)=Y5OLD
      Y(6)=Y6OLD
      VMEAS=VMOLD
      CONCNT=CONCNL
      INCR=INCR*0.5D0
      K=K+1
      GO TO 40
C
C                 NEXT PULSE HAS OCCURRED WITH REASONABLE
C                 ACCURACY.NOW SPEED IS DETERMINED
C
   38 COUNT=IDINT(CONCNT+UCOUNT)
      COUNT1=IABS(COUNT-RCOUNT)
      RCOUNT=LCOUNT
      LCOUNT=COUNT
      PERIOD=TIME-LCTIME
      LCTIME=TIME
      NCLOCK=IDINT(PERIOD*OSCF)
C
C                 THIS CHECK IS ONLY IMPORTANT AT STARTUP.
C                 THE FIRST ARRIVING PULSE CANNOT BE USED FOR
C                 SPEED MEASUREMENT.
C
  307 IF(COUNT1.NE.2)GO TO 308
      VMEAS=DP*OSCF/NCLOCK*DIR-XBIAS/TIME
      VDOT=(VMEAS-VOLD)/PERIOD
      VOLD=VMEAS
      ERROR1=V2-VMEAS
      N=N+1
      NPTS=N-2
  308 INCR=DELT
C
C
C                 END OF TACHO ROUTINE
C
C
  301 IF(TIME.LE.DNEXT)GO TO 310
      DNEXT=DPLOT+DNEXT
      WRITE(6,200)TIME,PERIOD,TNET2,V2,VMEAS,X2,ERROR1,COUNT
      IF(NPRINT.NE.75)GO TO 302
      WRITE(6,260)
      NPRINT=0
  302 NPRINT=NPRINT+1
  310 IF(TIME.LE.PLOTD)GO TO 320
      PLOTD=PLOTD+PLOTT
```

```
      J=J+1
      GO TO (400,401,402),CONECT
  400 XA(J)=X2
      VA(J)=V2
      TA(J)=TIME
      GO TO 320
  401 XD(J)=X2
      VD(J)=V2
      TD(J)=TIME
      GO TO 320
  402 TS(J)=TIME
      VS(J)=V2
      XS(J)=X2
  320 ERROR1=V2-VMEAS
      IF(TIME.GT.FIN)GO TO 600
   90 CONTINUE
C
C
  600 PTS=J-2
      WRITE(6,210)I
      WRITE(6,220)J
      WRITE(6,240)N
      WRITE(6,230)PTS
      WRITE(6,270)K
      WRITE(6,260)
  100 FORMAT('1',//////,5X,'IMPORTANT PARAMETER VALUES ARE
     1',//)
  101 FORMAT(6A8)
  102 FORMAT(5D10.5)
  103 FORMAT(E10.4,2I5)
  104 FORMAT(///,4X,'INITIAL TIME= ',F10.4,2X,'LARGEST INCRE
     1MENT= ',F10.8)
  105 FORMAT(//,4X,'FINAL   TIME= ',F10.4,2X,'PLOT INTERVAL=
     1     ',F10.6)
  106 FORMAT(//,4X,'FEEDBACK TAKEN FROM ANALOG TACHO MEASURE
     1MENT')
  107 FORMAT(//,4X,'FEEDBACK TAKEN FROM DIGITAL TACHO MEASUR
     1EMENT')
  109 FORMAT(//,4X,'FEDBACK TAKEN FROM SAMPLER')
  110 FORMAT(5X,4HGAIN,7X,5HFGAIN,6X,6HKTACHO,//)
  120 FORMAT(F12.1,2F12.6)
  130 FORMAT(////,5X,'INITIAL CONDITIONS ARE',//)
  140 FORMAT(5X,3HV20,8X,3HX20,8X,5HVMEAS,6X,6HFBTIME,//)
  150 FORMAT(4F12.5,//)
  180 FORMAT('1','      THE TACHO COUNT VALUES AND SPEEDS ARE
     1')
  190 FORMAT('0',8X,4HTIME,9X,6HPERIOD,9X,5HTNET2,9X,2HV2,
     112X,5HVMEAS,9X,2HX2,11X,6HERROR1,14X,5HCOUNT)
  200 FORMAT( 7D15.7,I12)
  210 FORMAT('0','      THE TOTAL NUMBER OF ITERATIONS IS    I=
     1   ',I6,//)
  220 FORMAT('0','      THE SIZE OF THE STORAGE ARRAYS IS   J =
     1 ',I6,//)
  230 FORMAT('0','      THE NUMBER OF POINTS PLOTTED IS   PTS=
     1',I6)
  240 FORMAT('0','    THE TOTAL NUMBER OF POINTS FOR DIGITAL
     1TACHO IS   N= ',I6,//)
  250 FORMAT(//,4X,'ERROR STOP')
  260 FORMAT('1')
  270 FORMAT(//,'     THE TOTAL NUMBER OF INTEGRATIONS IS  K=
     1 ',I12)
C
C
      GO TO (1,501,1),CONECT
  500 ITB(1)=0
      CALL DRAWP(PTS,TA,VA,ITB,RTB)
      CALL DRAWP(PTS,TA,XA,ITB,RTB)
      GO TO 1
  501 ITB(1)=1
      CALL DRAWP(PTS,TA,VA,ITB,RTB)
      ITB(1)=2
      CALL DRAWP(PTS,TS,VS,ITB,RTB)
```

```
      ITB(1)=3
      CALL DRAWP(PTS,TD,VD,ITB,RTB)
      READ(5,101)TITLE
      ITB(1)=1
      CALL DRAWP(PTS,TA,XA,ITB,RTB)
      ITB(1)=2
      CALL DRAWP(PTS,TS,XS,ITB,RTB)
      ITB(1)=3
      CALL DRAWP(PTS,TD,XD,ITB,RTB)
      GO TO 1
  502 ITB(1)=0
      CALL DRAWP(PTS,TS,XS,ITB,RTB)
      CALL DRAWP(PTS,TS,VS,ITB,RTB)
      GO TO 1
C
  999 STOP
      END
```

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of a digital tachometer
and its effects on feed-
back control systems.